Chapter 1 – Number

Q1 (a) $2 + 4 \div (2 - 1) = 6$

(b) $(2 + 4) \div 2 - 1 = 2$

(c) $2 + (4 \div 2) - 1 = 3$

Q2 (a) **(i)** 340

$$\begin{array}{r|r} 2 & 340 \\ \hline 2 & 170 \\ \hline 5 & 85 \\ \hline 17 & 17 \\ \hline & 1 \end{array}$$

340 as a product of primes is:
$2^2 \times 5 \times 17$

(ii) 1950

$$\begin{array}{r|r} 2 & 1950 \\ \hline 3 & 975 \\ \hline 5 & 325 \\ \hline 5 & 65 \\ \hline 13 & 13 \\ \hline & 1 \end{array}$$

1950 as a product of primes is:
$2 \times 3 \times 5^2 \times 13$

(iii) 2100

$$\begin{array}{r|r} 2 & 2100 \\ \hline 2 & 1050 \\ \hline 3 & 525 \\ \hline 5 & 175 \\ \hline 5 & 35 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

2100 as a product of primes is:
$2^2 \times 3 \times 5^2 \times 7$

(b) **(i)** 34, 48 and 52:

Factors of 34 are: 1, **2**, 17, 34.

Factors of 48 are: 1, **2**, 3, 4, 6, 8, 12, 16, 24, 48.

Factors of 52 are: 1, **2**, 4, 13, 26, 52.

Therefore the HCF is **2**.

(ii) 70, 105 and 126:

Factors of 70 are: 1, 2, 5, **7**, 10, 14, 35, 70.

Factors of 105 are: 1, 3, 5, **7**, 15, 21, 35, 105.

Factors of 126 are: 1, 2, 3, 6, **7**, 9, 14, 18, 21, 42, 63, 126.

Therefore the HCF is **7**.

(iii) 248, 320 and 364:

Factors of 248 are: 1, 2, **4**, 8, 31, 62, 124, 248.

Factors of 320 are: 1, 2, **4**, 5, 8, 10, 16, 20, 32, 40, 64, 80, 160, 320.

Factors of 364 are: 1, 2, **4**, 7, 13, 14, 26, 28, 52, 91, 182, 364.

Therefore the HCF is **4**.

Q3

Number	Natural	Integer	Rational	Irrational	Real
$\frac{1}{6}$			✓		✓
$\frac{\pi}{6}$				✓	✓
6	✓	✓	✓		✓
−6		✓	✓		✓
$\sqrt{6}$				✓	✓

Q4 • Find the prime factors of each number.

$$\begin{array}{r|l} 2 & 234 \\ \hline 3 & 117 \\ \hline 3 & 39 \\ \hline 13 & 13 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 156 \\ \hline 2 & 78 \\ \hline 3 & 39 \\ \hline 13 & 13 \\ \hline & 1 \end{array}$$

• Write each number as a product of its prime factors.

$234 = 2 \times 3^2 \times 13$

$156 = 2^2 \times 3 \times 13$

• Choose the common prime factors with the lowest power.

$234 = \boxed{2} \times 3^2 \times \boxed{13}$

$156 = 2^2 \times \boxed{3} \times 13$

The common prime factors with the lowest powers are: 2, 3 and 13.

• To get the HCF, multiply the common prime factors with the lowest powers.

HCF $= 2 \times 3 \times 13 = 78$

This means that 78 cm is the length of one of the sides of the largest possible square that can be cut out.

Q5 (a) **(i)** 4, 6 and 8:

Multiples of 4 are 4, 8, 12, 16, 20, 24, 28, 32, …

Multiples of 6 are 6, 12, 18, 24, 30, 36, 42, …

Multiples of 8 are 8, 16, 24, …

The LCM of 4, 6 and 8 is 24.

(ii) 12, 18 and 24:

Multiples of 12 are 12, 24, 36, 48, 60, 72, 84, 96, …

Multiples of 18 are 18, 36, 54, 72, 90, 108, …

Multiples of 24 are 24, 48, 72, …

The LCM of 12, 18 and 24 is 72.

(iii) $8 = 2 \times 2 \times 2 = \boxed{2^3}$

$7 = 1 \times \boxed{7}$

$21 = \boxed{3} \times 7$

The common prime factors to their highest powers are: 2^3, 3 and 7.

The LCM of 8, 7 and 21 is $2^3 \times 3 \times 7 = 168$.

(b) • Write each number as a product of its prime factors.

$12 = 2 \times 2 \times 3 = 2^2 \times 3$

$8 = 2 \times 2 \times 2 = 2^3$

$7 = 7 \times 1$

- Write down each prime factor to its highest power as it appears as a prime factor of any of the numbers in the question.

  $12 = 2 \times 2 \times 3 = 2^2 \times \boxed{3}$

  $8 = 2 \times 2 \times 2 = \boxed{2^3}$

  $7 = \boxed{7} \times 1$

  The common prime factors to its highest power are: 2^3, 3 and 7.

- To get the LCM, multiply all these prime factors.
  LCM is $2^3 \times 3 \times 7 = 168$.

  This means that Ciara would have to buy 168 plates.

Q6 (a) $3 + 2 \times 4 \div 2(5 - 3)^2 + (4 + 2 \times 3)$ Step 1: Brackets.

$= 3 + 2 \times 4 \div 2(2)^2 + (4 + (2 \times 3))$

$= 3 + 2 \times 4 \div 2(2)^2 + (4 + 6)$ Step 2: Indices (powers).

$= 3 + 2 \times 4 \div 2(4) + (4 + 6)$ Steps 3 and 4: Multiplication and division.

$= 3 + 1 + 4 + 6$ Steps 5 and 6: Addition and subtraction.

$= 14$

(b) $\dfrac{3(1+4)^2 - 5}{9 - 4}$ Step 1: Brackets.

$= \dfrac{3(5)^2 - 5}{9 - 4}$ Step 2: Indices (powers).

$= \dfrac{3(25) - 5}{9 - 4}$

$= \dfrac{75 - 5}{9 - 4}$ Steps 3 and 4: Multiplication and division.

$= \dfrac{70}{5}$ Steps 5 and 6: Addition and subtraction.

$= 14$ Simplify.

(c) $\left(\dfrac{3}{2} \div \dfrac{2}{3}\right)^2 - \dfrac{3}{4} \times \dfrac{5}{2} + \dfrac{1}{8}$ Step 1: Brackets.

$= \left(\dfrac{3}{2} \times \dfrac{3}{2}\right)^2 - \dfrac{3}{4} \times \dfrac{5}{2} + \dfrac{1}{8}$ Step 2: Indices (powers).

$= \left(\dfrac{9}{4}\right)^2 - \dfrac{3}{4} \times \dfrac{5}{2} + \dfrac{1}{8}$

$= \dfrac{81}{16} - \dfrac{3}{4} \times \dfrac{5}{2} + \dfrac{1}{8}$ Steps 3 and 4: Multiplication and division.

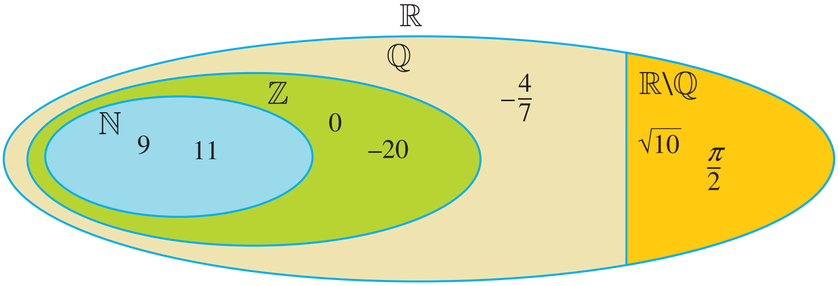
$= \dfrac{81}{16} - \left(\dfrac{3}{4} \times \dfrac{5}{2}\right) + \dfrac{1}{8}$

$= \dfrac{81}{16} - \left(\dfrac{15}{8}\right) + \dfrac{1}{8}$ Steps 5 and 6: Addition and subtraction (LCM 16).

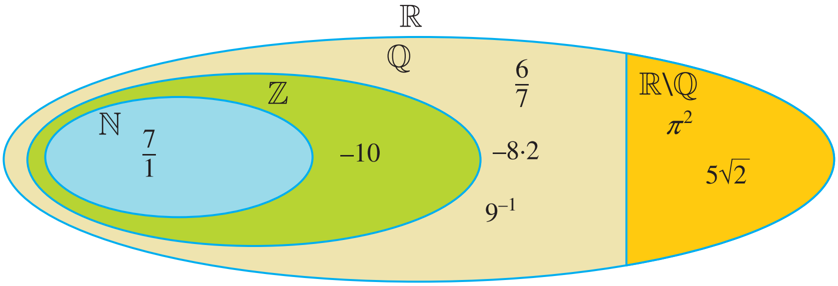
$= \dfrac{81}{16} - \dfrac{14}{8}$

$= \dfrac{53}{16} = 3\dfrac{5}{16}$

Q7 (a) (i)



(ii)



(b) (i) $\sqrt{150}$ — Get factors such that one of the factors is a square number.

$= \sqrt{25 \times 6}$

$= \sqrt{25} \times \sqrt{6}$ — Simplify $\sqrt{25}$.

$= 5\sqrt{6}$

(ii) $\sqrt{112} - \sqrt{63} + \sqrt{28}$ — Get factors such that one of the factors is a square number in each term.

$= \sqrt{16 \times 7} - \sqrt{9 \times 7} + \sqrt{4 \times 7}$ — Simplify $\sqrt{16}$, $\sqrt{9}$ and $\sqrt{4}$.

$= 4\sqrt{7} - 3\sqrt{7} + 2\sqrt{7}$ — Simplify again.

$= 3\sqrt{7}$

(iii) $\sqrt{2}(4 + 2\sqrt{2})$

$= \sqrt{2}(4) + \sqrt{2}\left(2\sqrt{2}\right)$ — Simplify using $\sqrt{2} \times \sqrt{2} = 2$.

$= 4\sqrt{2} + 4$

Q8 (a) (i) $3{\cdot}5$, $\frac{17}{5} = 3{\cdot}4\ldots$ and $2\sqrt{3} = 3{\cdot}4641$ — Write as decimals.

$\frac{17}{5}, 2\sqrt{3}, 3{\cdot}5$ — Reorder.

(ii) $2\pi = 6{\cdot}283\ldots$, $3\sqrt{5} = 6{\cdot}708\ldots$ and

$\frac{20}{3} = 6{\cdot}667$ — Write as decimals.

$2\pi, \frac{20}{3}, 3\sqrt{5}$ — Reorder.

(b) (i) $\frac{9}{10}+\frac{5}{6}$ (LCM = 30)

$=\frac{3(9)}{30}+\frac{5(5)}{30}$

$=\frac{27}{30}+\frac{25}{30}$

$=\frac{27+25}{30}$

$=\frac{52}{30}=1\frac{22}{30}=1\frac{11}{15}$

(ii) $\frac{9}{10}-\frac{5}{6}$ (LCM = 30)

$=\frac{3(9)}{30}-\frac{5(5)}{30}$

$=\frac{27}{30}-\frac{25}{30}$

$=\frac{27-25}{30}$

$=\frac{2}{30}=\frac{1}{15}$

(iii) $\frac{9}{10}\times\frac{5}{6}=\frac{9\times 5}{10\times 6}=\frac{45}{60}=\frac{3}{4}$

(iv) $\frac{9}{10}\div\frac{5}{6}=\frac{9}{10}\times\frac{6}{5}=\frac{54}{50}=\frac{27}{25}$

Chapter 2 – Sets

Q1 (a) {}, {10}, {20}, {10, 20}

(b) From observation of the Venn diagram:

(i) $P=\{1, 2, 3, 4\}$

(ii) $Q=\{6, 7, 8, 9\}$

(iii) $P\cup Q=\{1, 2, 3, 4, 6, 7, 8, 9\}$

(iv) $P\cap Q=\{\ \}$

(v) $P\setminus Q=\{1, 2, 3, 4\}$

(vi) $Q\setminus P=\{6, 7, 8, 9\}$

(vii) $P'=\{5, 6, 7, 8, 9, 10\}$

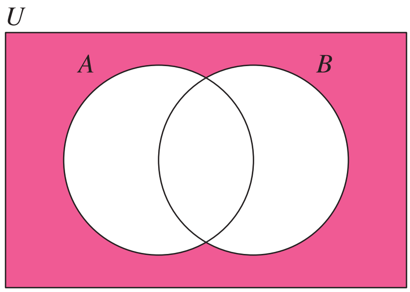
(viii) $Q'=\{1, 2, 3, 4, 5, 10\}$

(ix) $(P\cap Q)'=\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

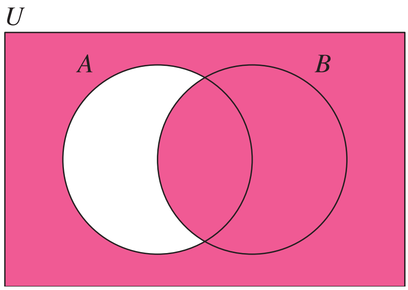
(x) $(Q\setminus P)'=\{1, 2, 3, 4, 5, 10\}$

Q2 (a) {}, {red}, {green}, {blue}, {red, green}, {red, blue}, {green, blue}, {red, green, blue}

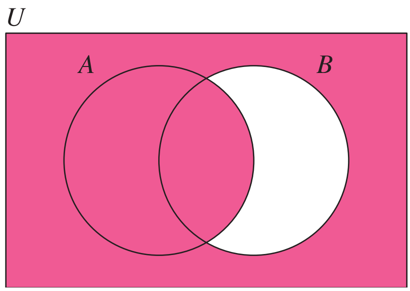
(b) (i) $(A\cup B)'$



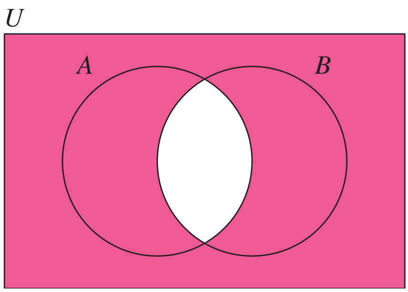
(iii) $(A\setminus B)'$



(ii) $U\setminus(B\setminus A)$



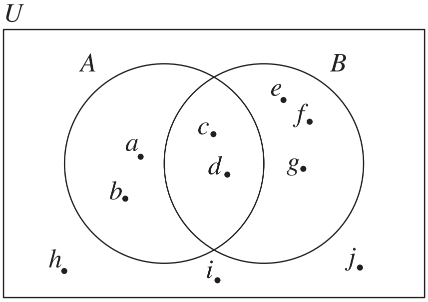
(iv) $U\setminus(A\cap B)$



(c) $U = \{a, b, c, d, e, f, g, h, i, j\}$

$A = \{a, b, c, d\}$

$B = \{c, d, e, f, g\}$



(i) $A' = \{e, f, g, h, i, j\}$

(ii) $U \setminus (A \cap B) = \{a, b, e, f, g, h, i, j\}$

(iii) $(A \cup B)' = \{h, i, j\}$

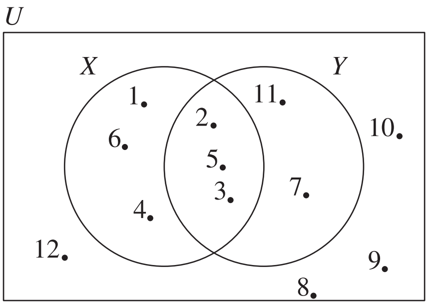
(iv) $(A \setminus B)' = \{c, d, e, f, g, h, i, j\}$

Q3 (a) $U = \{x \mid x < 13, x \in \mathbb{R}\}$

$X = \{x \mid x < 7, x \in \mathbb{R}\}$

$Y = \{x \mid x$ is one of the first five prime numbers$\}$

(b)



(i) $\#(X \cup Y) = 8$

(ii) $\#(X \cap Y) = 3$

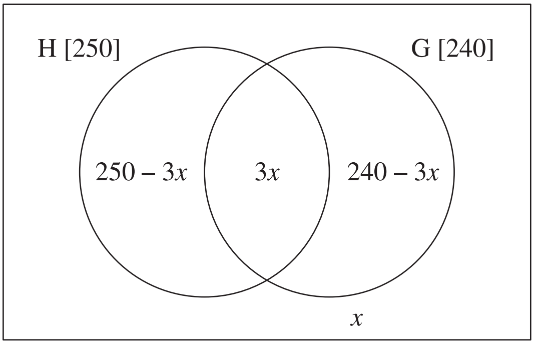
(iii) $\#Y' = 7$

(iv) $\#(X \cup Y)' = 4$

(v) $\#(X \setminus Y) = 3$

(vi) $\#(Y \setminus X) = 2$

(c) **(i)**



(ii) $\#U = 430$

$\#U = [250 - 3x] + [3x] + [240 - 3x] + [x] = 430$

$490 - 2x = 430$

$490 - 430 = 2x$

$60 = 2x$

$\therefore\ x = 30$

30 students study neither History nor Geography.

(d) **(i)** Let $\#(E \cup F)' = x$, the number of students who have not been to either country. Now complete the Venn diagram as follows:

Find the cardinal number for the members that are common to both sets.

$\#(E \cap F) = 13$

Find the cardinal numbers for the members that remain in each set.

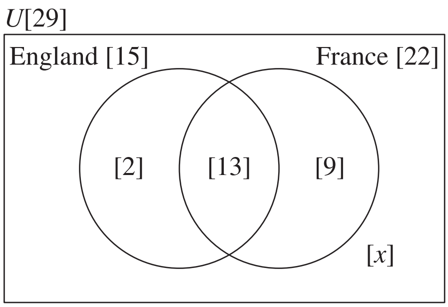
$\#(E \backslash F) = 15 - 13 = 2$

$\#(F \backslash E) = 22 - 13 = 9$

Find the cardinal number for any of the remaining members listed in the universal set that have not been used.

$\#(E \cup F)' = x$

Fill in all information into a Venn diagram.



(ii) Solve for x:

$\#(E \cup F)' = x$

$\#U = 29$

$\#U \backslash (E \cup F) = \#(E \cup F)' = x$

$2 + 13 + 9 + x = 29$

$24 + x = 29$

$\therefore x = 5$

5 students in the class have not been to either country.

Q4 **1** Find the cardinal number for the members that are common to all three sets.

$\#(T \cap C \cap H) = x$

2 Find the cardinal numbers for the members that are common to two sets.

$\#(T \cap C) \setminus H = 57 - x$

$\#(C \cap H) \setminus T = 32 - x$

$\#(T \cap H) \setminus C = 29 - x$

3 Find the cardinal numbers for the members that are unique to each set.

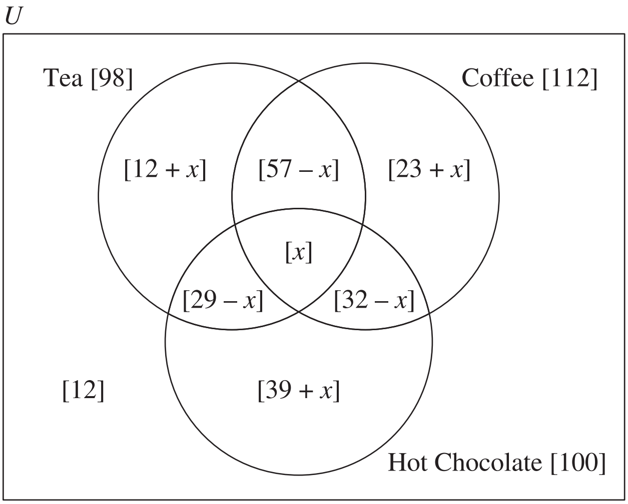
$\#T \setminus (C \cup H) = 98 - [57 - x + x + 29 - x] = 12 + x$

$\#C \setminus (T \cup H) = 112 - [57 - x + x + 32 - x] = 23 + x$

$\#H \setminus (T \cup C) = 100 - [29 - x + x + 32 - x] = 39 + x$

4 Find the cardinal number for any remaining members listed in the universal set that have not been used.

$\#U \setminus (T \cup C \cup H) = 12$



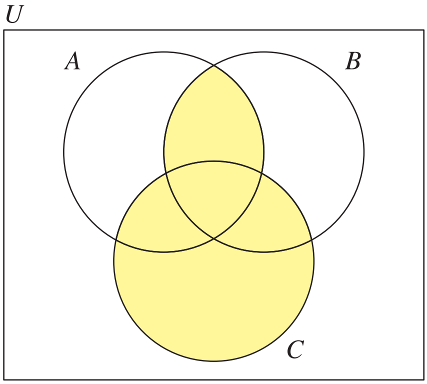
$\#U = 220$

$\#U = [12 + x] + [57 - x] + [23 + x] + [29 - x] + [x] + [32 + x] + [39 + x] + 12$

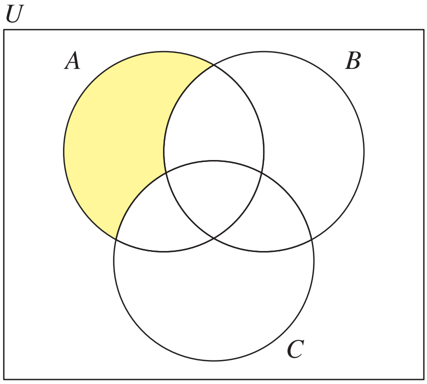
$204 + x = 220$

$\therefore x = 16$

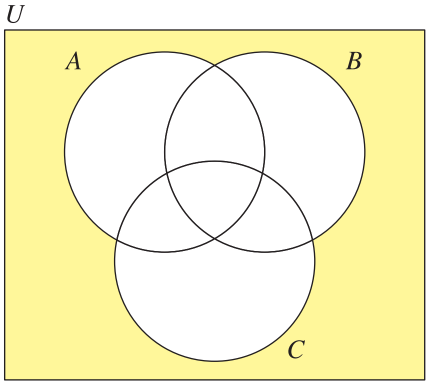
Q5 (a) (i) $(A \cap B) \cup C$



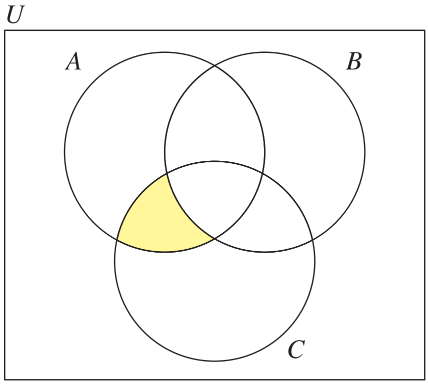
(ii) $(A \setminus B) \setminus C$



(iii) $(A \cup C)' \setminus B$



(iv) $(A \setminus B) \cap (C \setminus B)$



(b) 1 Find the cardinal number for the members that are common to both sets

$\#(B \cap C) = x$

2 Find the cardinal numbers for the members that remain in each set.

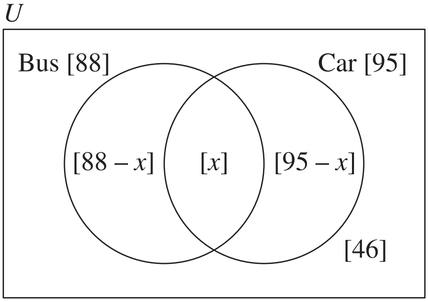
$\#(B \setminus C) = 88 - x$

$\#(C \setminus B) = 95 - x$

3 Find the cardinal number for any of the remaining members listed in the universal set that have not been used.

$\#U \setminus (B \cup C) = \#(B \cup C)' = 46$

4 Fill all information into a Venn diagram.



(i) To get the value for the smallest number of students that could have taken part in the survey, $\#(B \cup C)$, we must maximise the value of $\#(B \cap C)$.

$\therefore \#(B \cap C) = 88$ This is the largest possible value for x.

The total number of students in the survey is given by:

$\#U = [88 - x] + [x] + [95 - x] + [46]$

$\#U = 229 - x$

Given that $\#(B \cap C) = x = 88$,

$\#U = 229 - 88 = 141$

The smallest number of students that could have taken part in the survey is 141.

(ii) To get the value for the largest number of students that could have taken part in the survey, $\#(B \cup C)$, we must minimise the value of $\#(B \cap C)$.

$\therefore \#(B \cap C) = x = 0$ This is the smallest possible value for x.

The total number of students in the survey is given by:

$\#U = [88 - x] + [x] + [95 - x] + [46]$

$\#U = 229 - x$

Given that $\#(B \cap C) = x = 0$,

$\#U = 229 - 0 = 229$

The largest number of students that could have taken part in the survey is 229.

Chapter 3 – Applied Arithmetic 1

Q1 (a) Convert to the same unit:

38 cents

€2·50 = 250 cents

$\frac{38}{250} \times 100\% = 15{\cdot}2\% = 15\%$ to the nearest whole number

(b) Convert to the same unit:

50 seconds

3 minutes = $3 \times 60 = 180$ seconds

$\frac{50}{180} \times 100\% = 27{\cdot}\dot{7}\% = 28\%$ to the nearest whole number

(c) $\frac{15}{75} \times 100\% = 20\%$

Q2 (a) Increase in staff = 1450 − 1200 = 250 staff

$\% \text{ increase} = \frac{250}{1200} \times 100\%$

$= 20{\cdot}8\dot{3}\% = 20{\cdot}8\%$ to 1 d.p.

(b) '12% off' means a decrease of 12%.

100% − 12% = 88% = 0·88

€22 × 0·88 = €19·36

Therefore, the new price of a student's haircut is €19·36.

(c) 21% = 42 cupcakes

$1\% = \frac{42}{21} = 2$ cupcakes

100% = 2 × 100 = 200 cupcakes

Therefore, Ciara had 200 cupcakes at the start of the day.

Q3 (a) Estimate:

$\frac{(10{\cdot}21)^2}{4{\cdot}91}(9{\cdot}98) - \sqrt{3{\cdot}81}(6{\cdot}13)^2$

Round each decimal to the nearest whole number.

$\approx \frac{(10)^2}{5}(10) - \sqrt{4}(6)^2$

$= \frac{100}{5}(10) - 2(36)$

Follow the rules of BIRDMAS.

$= 20(10) - 72 = 200 - 72 = 128$

Evaluate correct to one decimal place:

$\frac{(10{\cdot}21)^2}{4{\cdot}91}(9{\cdot}98) - \sqrt{3{\cdot}81}(6{\cdot}13)^2$

Follow the rules of BIRDMAS.

$= \frac{104{\cdot}2441}{4{\cdot}91}(9{\cdot}98) - (1{\cdot}9519...)\,(37{\cdot}5769)$

$= 21{\cdot}231(9{\cdot}98) - (73{\cdot}3472)$

$= 211{\cdot}88538 - 73{\cdot}3472 = 138{\cdot}53818 = 138{\cdot}5$ to 1 d.p.

(b) Estimate:

$\frac{\sqrt{35{\cdot}91}(6{\cdot}16) + 17{\cdot}94 - (4{\cdot}12)^2}{(3{\cdot}89)^3 - 7{\cdot}32}$

Round each decimal to the nearest whole number.

$\approx \frac{\sqrt{36}(6) + 18 - (4)^2}{(4)^3 - 7}$

$= \dfrac{6(6) + 18 - 16}{64 - 7}$

Follow the rules of BIRDMAS.

$= \dfrac{36 + 18 - 16}{64 - 7}$

$= \dfrac{38}{57} = \dfrac{2}{3} = 0{\cdot}\dot{6} \approx 0{\cdot}7 \approx 1$

Evaluate correct to two decimal places:

$\dfrac{\sqrt{35{\cdot}91}(6{\cdot}16) + 17{\cdot}94 - (4{\cdot}12)^2}{(3{\cdot}89)^3 - 7{\cdot}32}$

Follow the rules of BIRDMAS.

$= \dfrac{5{\cdot}9925(6{\cdot}16) + 17{\cdot}94 - (16{\cdot}9744)}{58{\cdot}8639 - 7{\cdot}32}$

$= \dfrac{36{\cdot}9138 + 17{\cdot}94 - 16{\cdot}9744}{51{\cdot}5439}$

$= \dfrac{37{\cdot}8794}{51{\cdot}5439} = 0{\cdot}73489\ldots = 0{\cdot}73$ to 2 d.p.

Q4 (a) €360 in the ratio 5 : 11

Total parts = 5 + 11 = 16 parts

Each portion of the ratio:

$\dfrac{5}{16} \times €360 = €112{\cdot}50$

$\dfrac{11}{16} \times €360 = €247{\cdot}50$

(b) 4494 cm in the ratio 3 : 5 : 13

Total parts = 3 + 5 + 13 = 21 parts

Each portion of the ratio:

$\dfrac{3}{21} \times 4494 = 642$ cm

$\dfrac{5}{21} \times 4494 = 1070$ cm

$\dfrac{13}{21} \times 4494 = 2782$ cm

(c) 3·120 kg = 3120 g

Total parts = 3 + 6 + 11 = 20 parts

Each portion of the ratio:

$\dfrac{3}{20} \times 3120 = 468$ g

$\dfrac{6}{20} \times 3120 = 936$ g

$\dfrac{11}{20} \times 3120 = 1716$ g

Q5 Ratio = 10 : 5 : 3

Total parts = 10 + 5 + 3 = 18 parts

Find the value of one part:

3rd prize is $\frac{3}{18}$ = €1539

$\therefore \frac{1}{18} = \frac{€1539}{3} = €513$

Total prize fund is $\frac{18}{18}$ = €513 × 18 = €9234.

Q6 (a) Ratio = $3\frac{1}{2} : 2\frac{1}{2}$

Total parts = $3\frac{1}{2} + 2\frac{1}{2} = 6$

Peter's share is €35 000, so $3\frac{1}{2}$ parts = €35 000

1 part = $\frac{35\,000}{3\frac{1}{2}}$ = €10 000

Total prize fund = €10 000 × 6 = €60 000

(b) Estimate:

$$\frac{\sqrt{(7{\cdot}17)^2 + 14{\cdot}59}}{8{\cdot}29 - 1{\cdot}64 \times 2{\cdot}23} = \frac{\sqrt{(7)^2 + 15}}{8 - 2 \times 2}$$

$$\approx \frac{\sqrt{64}}{4} = \frac{8}{4} = 2$$

Correct to one decimal place:

$$\frac{\sqrt{(7{\cdot}17)^2 + 14{\cdot}59}}{8{\cdot}29 - 1{\cdot}64 \times 2{\cdot}23} \approx \frac{\sqrt{51{\cdot}4089 + 14{\cdot}59}}{8{\cdot}29 - 3{\cdot}6572}$$

$$= \frac{\sqrt{65{\cdot}9989}}{4{\cdot}6328} = \frac{8{\cdot}123970704}{4{\cdot}6328} = 1{\cdot}7535... = 1{\cdot}8\text{, correct to 1 d.p.}$$

Q7 (a) Distance travelled = 305 km

Time taken = 3 hours 15 minutes = 3 hours and $\frac{15}{60}$ minutes = 3·25 h

Speed = $\frac{\text{distance}}{\text{time}} = \frac{305}{3{\cdot}25}$ = 93·85 ≈ 94 km/h

(b) Distance travelled = 305 km

Reduced speed = 94 km/h − 16 km/h = 78 km/h

Time taken at this new speed:

Time = $\frac{\text{distance}}{\text{speed}} = \frac{305}{78} = 3{\cdot}9\dot{1}0256\dot{4}$

$= 3{\cdot}91$ h $= 3$ hours and $(0{\cdot}91 \times 60)$ minutes $= 3$ hours $54{\cdot}6$ minutes
≈ 3 hours 55 minutes

Extra time travelling on the return journey:

3 hours 55 minutes – 3 hours 15 minutes = 40 minutes

(c) 10:40 am + 3 h 55 mins = 14:35 = 2:35 pm

Q8 Convert units:

88 km = 88 × 1000 = 88 000 m

1 hour = 60 × 60 seconds = 3600 seconds

Speed $= \frac{\text{distance}}{\text{time}} = \frac{88\,000}{3600} = 24{\cdot}\dot{4}$ m/s $\approx 24{\cdot}444$ m/s

Chapter 4 – Algebra 1: Algebraic Expressions

Q1 In the quadratic expression $4x^2 + 12x + 9$:

(a) the coefficient of x^2 is 4

(b) the coefficient of x is +12

(c) the constant is +9.

Q2 (a) When $k = 4$: $-4(3k + 5) = -4(3(4) + 5) = -4(12 + 5) = -4(17) = -68$.

(b) When $d = 6$: $\frac{d-6}{7} = \frac{(6)-6}{7} = \frac{0}{7} = 0$.

(c) When $h = 2$: $-3(h - 7)^2 = -3((2) - 7)^2 = -3(-5)^2 = -3(25) = -75$.

(d) When $g = -1$: $-4(-7 - 6g + g^2) = -4(-7 - 6(-1) + (-1)^2)$
$= -4(-7 + 6 + 1) = -4(0) = 0$.

Q3 (a) When $p = 5$ and $r = -3$: $\frac{p^2 - (r-2)}{6} = \frac{(5)^2 - ((-3) - 2)}{6} = \frac{25 - (-5)}{6}$
$= \frac{25 + 5}{6} = \frac{30}{6} = 5$.

(b) When $a = 7$ and $b = 11$: $\frac{(a+b)^2 - 27b}{a - 3b + 20} = \frac{((7) + (11))^2 - 27(11)}{(7) - 3(11) + 20}$
$= \frac{(18)^2 - 297}{7 - 33 + 20} = \frac{324 - 297}{-6} = \frac{27}{-6} = -\frac{9}{2}$.

Q4 (a) $a + b - c + 2b + 7c - 5 = a + 3b + 6c - 5$

(b) $x^2 - 7x + 2x^2 - 11x + 13 = 3x^2 - 18x + 13$

(c) $xy + y^2 - 2y - 7x - 17xy - 4y^2 = -16xy - 3y^2 - 2y - 7x$

(d) $d^3 - 13 - 2d - 7d^2 + d + 7 = d^3 - 7d^2 - d - 6$

Q5 (a) $2(b - 3) = 2b - 6$

(b) $x(x - 2) + 3(2x + 9) = x^2 - 2x + 6x + 27 = x^2 + 4x + 27$

(c) $2n(n - 3) + 7m(5n - 7) = 2n^2 - 6n + 35mn - 49m$

(d) $x(x^2 + x - 2) + 3(x^2 - 7x + 1) = x^3 + x^2 - 2x + 3x^2 - 21x + 3 = x^3 + 4x^2 - 23x + 3$

Q6 (a) $(2x+3)^2 = (2x)^2 + 2(2x)(3) + (3)^2 = 4x^2 + 12x + 9$

Alternative method: $(2x+3)(2x+3) = 2x(2x+3) + 3(2x+3)$
$= 4x^2 + 6x + 6x + 9 = 4x^2 + 12x + 9$

(b) $(7x+8y)^2 = (7x)^2 + (2)(7x)(8y) + (8y)^2 = 49x^2 + 112xy + 64y^2$

Q7 (a) $(4x-3)(4x+3) = (4x)^2 - (3)^2 = 16x^2 - 9$

(b) $(2a-7b)(2a+7b) = (2a)^2 - (7b)^2 = 4a^2 - 49b^2$

Q8 (a) $(a-2)(a+5) = a(a+5) - 2(a+5) = a^2 + 5a - 2a - 10 = a^2 + 3a - 10$

(b) $(p-5)(p+3) = p(p+3) - 5(p+3) = p^2 + 3p - 5p - 15 = p^2 - 2p - 15$

(c) $(6x-3)(7-2x) = 6x(7-2x) - 3(7-2x) = 42x - 12x^2 - 21 + 6x = -12x^2 + 48x - 21$

(d) $(7x-1)(3x-2) = 7x(3x-2) - 1(3x-2) = 21x^2 - 14x - 3x + 2 = 21x^2 - 17x + 2$

Q9 (a) $(x-3)(1-2x+y) = x(1-2x+y) - 3(1-2x+y) = x - 2x^2 + xy - 3 + 6x - 3y$

$= -2x^2 + xy - 3 + 7x - 3y$

(b) $(2x-3y)(-2x^2+9xy-7y^2) = 2x(-2x^2+9xy-7y^2) - 3y(-2x^2+9xy-7y^2)$
$= -4x^3 + 18x^2y - 14xy^2 + 6x^2y - 27xy^2 + 21y^3 = -4x^3 + 21y^3 + 24x^2y - 41xy^2$

Chapter 5 – Algebra 2: Factorising

Q1 (a) $4a + 8b = 4(a+2b)$

(b) $2xy + 18x = 2x(y+9)$

(c) $a^2b - a^2b^2 = a^2b(1-b)$

(d) $5x^2y^3 - 20xy^2 = 5xy^2(xy-4)$

(e) $4pq^2 - 8pq - 12p^2q = 4pq(q-2-3p)$

(f) $4\pi r^2 - 8\pi r = 4\pi r(r-2)$

Q2 (a) $a - b - ax + bx = 1(a-b) - x(a-b) = (1-x)(a-b)$

(b) $c - d + cd - 1 = c + cd - 1 - d = c(1+d) - 1(1+d) = (c-1)(1+d)$

(c) $5x + ay + 5y + ax = 5x + ax + ay + 5y = x(5+a) + y(a+5) = (x+y)(a+5)$
Note: $5 + a = a + 5$

(d) $9a^2b + 3a^2 + 5b^2 + 15b^3 = 3a^2(3b+1) + 5b^2(1+3b) = (3a^2+5b^2)(1+3b)$
Note: $1 + 3b = 3b + 1$

(e) $10x^2 - 5x + 2xy - y = 5x(2x-1) + y(2x-1) = (5x+y)(2x-1)$

Q3 (a) $2p(a-b) - 1(a-b) = (2p-1)(a-b)$
Note: $-(a-b) = -1(a-b)$

(b) $3a(p-3q) - 2b(p-3q) = (3a-2b)(p-3q)$

(c) $7x(2y-7z) + 1(7z-2y) = 7x(2y-7z) - 1(-7z+2y) = (7x-1)(2y-7z)$
Note: $+1(7z-2y) = -1(-7z+2y)$

(d) $(3x-7y) - 4p(3x-7y) = 1(3x-7y) - 4p(3x-7y) = (1-4p)(3x-7y)$

Q4 (a) $x^2 - y^2 = (x - y)(x + y)$

(b) $25c^2 - 49d^2 = (5c)^2 - (7d)^2 = (5c - 7d)(5c + 7d)$

(c) $36a^2 - 9b^2 = (6a)^2 - (3b)^2 = (6a - 3b)(6a + 3b)$

(d) $1 - 9a^2b^2 = (1)^2 - (3ab)^2 = (1 - 3ab)(1 + 3ab)$

Q5 (a) $2x^2 - 32 = 2(x^2 - 16) = 2[(x^2) - (4)^2] = 2(x - 4)(x + 4)$

(b) $12a^2y^2 - 27b^2y^2 = 3(4a^2y^2 - 9b^2y^2) = 3[(2ay)^2 - (3by)^2]$
$= 3(2ay - 3by)(2ay + 3by)$

(c) $43^2 - 13^2 = (43 - 13)(43 + 13) = (30)(56) = 1680$

(d) $27^2 - 3^2 = (27 - 3)(27 + 3) = (24)(30) = 720$

Any method can be used to factorise the quadratic trinomials in **Q6** and **Q7**. The solutions here use the guide number method.

Q6 (a) $x^2 - 6x + 9$ — The GN is (1)(+9) = +9.

Factors of +9 which add to −6 are (−3)(−3).

$= x^2 - 3x - 3x + 9$ — Break up $-6x$ into $-3x - 3x$.

$= x(x - 3) - 3(x - 3)$ — Factorise using the grouping method.

$= (x - 3)(x - 3)$

$= (x - 3)^2$

(b) $c^2 - 2c + 1$ — The GN is (1)(+1) = +1.

Factors of +1 which add to −2 are (−1)(−1).

$= c^2 - c - c + 1$ — Break up $-2c$ into $-c - c$.

$= c(c - 1) - 1(c - 1)$ — Factorise using the grouping method.

$= (c - 1)(c - 1)$

$= (c - 1)^2$

(c) $x^2 + 2xy + y^2$ — The GN is (1)(+1) = +1.

Factors of +1 which add to +2 are (+1)(+1).

$= x^2 + xy + xy + y^2$ — Break up $+2xy$ into $+xy + xy$.

$= x(x + y) + y(x + y)$ — Factorise using the grouping method.

$= (x + y)(x + y)$

$= (x + y)^2$

(d) $x^2 - 6x - 16$ — The GN is (1)(−16) = −16.

Factors of −16 which add to −6 are (−8)(+2).

$= x^2 + 2x - 8x - 16$ — Break up $-6x$ into $+2x - 8x$.

$= x(x + 2) - 8(x + 2)$ — Factorise using the grouping method.

$= (x - 8)(x + 2)$

(e) $x^2 + 11x - 12$ — The GN is (1)(−12) = −12.

Factors of −12 which add to +11 are (−1)(+12).

$= x^2 - x + 12x - 12$ — Break up $+11x$ into $-x + 12x$.

$= x(x - 1) + 12(x - 1)$ — Factorise using the grouping method.

$= (x + 12)(x - 1)$

(f) $x^2 - x - 2$ — The GN is (1)(−2) = −2.

Factors of −2 which add to −1 are (+1)(−2).

$= x^2 + x - 2x - 2$ — Break up $-x$ into $+x - 2x$.

$= x(x + 1) - 2(x + 1)$ — Factorise using the grouping method.

$= (x - 2)(x + 1)$

Q7 (a) $2x^2 - 5x + 3$ — The GN is (2)(+3) = +6.

Factors of +6 which add to −5 are (−2)(+3).

$= 2x^2 - 2x - 3x + 3$ — Break up $-5x$ into $-2x - 3x$.

$= 2x(x - 1) - 3(x - 1)$ — Factorise using the grouping method.

$= (2x - 3)(x - 1)$

(b) $22x^2 - 13x + 1$ — The GN is (22)(+1) = +22.

Factors of +22 which add to −13 are (−11)(−2).

$= 22x^2 - 11x - 2x + 1$ — Break up $-13x$ into $-11x - 2x$.

$= 11x(2x - 1) - 1(2x - 1)$ — Factorise using the grouping method.

$= (11x - 1)(2x - 1)$

(c) $7x^2 + 19x - 6$ — The GN is (7)(−6) = −42.

Factors of −42 which add to +19 are (+21)(−2).

$= 7x^2 + 21x - 2x - 6$ — Break up $+19x$ into $+21x - 2x$.

$= 7x(x + 3) - 2(x + 3)$ — Factorise using the grouping method.

$= (7x - 2)(x + 3)$

(d) $6x^2 + 17x + 5$ — The GN is (6)(+5) = +30.

Factors of +30 which add to +17 are (+15)(+2).

$= 6x^2 + 2x + 15x + 5$ — Break up $+17x$ into $+2x + 15x$.

$= 2x(3x + 1) + 5(3x + 1)$ — Factorise using the grouping method.

$= (2x + 5)(3x + 1)$

Q8 (a) $6a^2b - 4ab^2 = 2ab(3a - 2b)$ Factorise using the HCF method.

(b) $36a^2 - 25b^2$

$= (6a)^2 - (5b)^2$

$= (6a - 5b)(6a + 5b)$

Factorise using the difference of two squares method.

(c) $xy - 7a + xa - 7y$

$= xy + xa - 7a - 7y$

$= x(y + a) - 7(a + y)$

$= (x - 7)(a + y)$

Factorise using the grouping method.
Remember that $(y + a) = (a + y)$.

Chapter 6 – Algebra 3: Algebraic Fractions and Long Division

Q1 (a) $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{12}{24}$

(b) (i) $\frac{12a^3b^5c^2}{3ab^6c^2} = \frac{4a^2}{b}$

(ii) $\frac{3a - 3b}{a - b} = \frac{3\cancel{(a - b)}}{\cancel{(a - b)}} = 3$

(iii) $\frac{(c - d)}{-(c - d)} = -1$

Q2 (a) $\frac{(x + 2)(x - 2)}{x^2 - 4x + 4} = \frac{(x + 2)\cancel{(x - 2)}}{(x - 2)\cancel{(x - 2)}} = \frac{(x + 2)}{(x - 2)}$

(b) $\frac{y - 5}{y^2 - 25} \times \frac{y + 5}{3} = \frac{\cancel{(y - 5)}\cancel{(y + 5)}}{3\cancel{(y - 5)}\cancel{(y + 5)}} = \frac{1}{3}$

(c) $\frac{(x + y^2)}{(x + y)} \times \frac{(x^2 - y^2)}{x} = \frac{(x + y^2)(x - y)\cancel{(x + y)}}{x\cancel{(x + y)}} = \frac{(x + y^2)(x - y)}{x}$

(d) $\frac{x^2 + ax - bx - ab}{bx + ab} \times \frac{ax^2 + a^2x}{ax} = \frac{(x - b)\cancel{(x + a)}}{b\cancel{(x + a)}} \times \frac{\cancel{ax}(x + a)}{\cancel{ax}}$

$= \frac{(x - b)}{b} \times \frac{(x + a)}{1} = \frac{(x - b)(x + a)}{b}$

Note: $x^2 + ax - bx - ab$
$= x(x + a) - b(x + a)$
$= (x - b)(x + a)$

Q3 (a) $\frac{a}{3b} \div \frac{a^2}{6abc} = \frac{a}{3b} \times \frac{6abc}{a^2} = \frac{6a^2bc}{3a^2b} = 2c$

(b) $\frac{9a^5}{x^4} \div \frac{3a^3}{3x^5} = \frac{9a^5}{x^4} \times \frac{3x^5}{3a^3} = \frac{27a^5x^5}{3a^3x^4} = 9a^2x$

(c) $\frac{x^2 - x}{2} \div \frac{x}{8} = \frac{x(x - 1)}{2} \times \frac{8}{x} = \frac{8x(x - 1)}{2x} = 4(x - 1) = 4x - 4$

Q4 (a) $\frac{1}{x}+\frac{3}{y}=\frac{y(1)+x(3)}{xy}=\frac{y+3x}{xy}$

(b) $2+\frac{1}{x}=\frac{x(2)+1(1)}{x}=\frac{2x+1}{x}$

(c) $\frac{x}{y}-\frac{y}{x}=\frac{x(x)-y(y)}{xy}=\frac{x^2-y^2}{xy}=\frac{(x-y)(x+y)}{xy}$

(d) $\frac{3c}{2}+\frac{5c}{6}=\frac{3(3c)+1(5c)}{6}=\frac{9c+5c}{6}=\frac{14c}{6}=\frac{7c}{3}$

(e) $\frac{14d}{3}-3d=\frac{14d-3(3d)}{3}=\frac{14d-9d}{3}=\frac{5d}{3}$

(f) $\frac{4x-1}{2}+\frac{3x-2}{5}=\frac{5(4x-1)+2(3x-2)}{10}=\frac{20x-5+6x-4}{10}=\frac{26x-9}{10}$

(g) $\frac{7x-4}{3}+\frac{2-3x}{5}=\frac{5(7x-4)+3(2-3x)}{15}=\frac{35x-20+6-9x}{15}=\frac{26x-14}{15}$

(h) $\frac{1}{3}(2b-1)+\frac{2}{5}(5b-3)=\frac{5(2b-1)+3(2)(5b-3)}{15}$

$=\frac{10b-5+6(5b-3)}{15}=\frac{10b-5+30b-18}{15}$

$=\frac{40b-23}{15}$

Note: $\frac{1}{3}(2b-1)=\frac{2b-1}{3}$

and $\frac{2}{5}(5b-3)=\frac{2(5b-3)}{5}$

(i) $\frac{x-1}{5}+\frac{3x-4}{3}-\frac{5x+3}{15}=\frac{3(x-1)+5(3x-4)-1(5x+3)}{15}$

$=\frac{3x-3+15x-20-5x-3}{15}=\frac{13x-26}{15}$

(j) $\frac{2x-1}{4}-\frac{x+1}{7}-\frac{5x-1}{28}=\frac{7(2x-1)-4(x+1)-1(5x-1)}{28}$

$=\frac{14x-7-4x-4-5x+1}{28}=\frac{5x-10}{28}$

Q5 (a)

$$\begin{array}{r|l} & x\;-2 \\ \hline x+1 & x^2-x-2 \\ & -x^2-x \\ \hline & \quad -2x-2 \\ & \quad +2x+2 \\ \hline & \qquad 0 \end{array}$$

Answer $= x-2$

(b)

$$\begin{array}{r|l} & 2x+3 \\ \hline x-5 & 2x^2-7x-15 \\ & -2x^2+10x \\ \hline & \quad +3x-15 \\ & \quad -3x+15 \\ \hline & \qquad 0 \end{array}$$

Answer $= 2x+3$

Q6 (a)

$$
\begin{array}{r}
x^2 - x - 6 \\
x - 4 \,\big)\overline{x^3 - 5x^2 - 2x + 24} \\
\underline{-x^3 + 4x^2 \qquad\qquad} \\
- \; x^2 - 2x \qquad \\
\underline{+ \; x^2 - 4x \qquad} \\
- 6x + 24 \\
\underline{+ 6x - 24} \\
0
\end{array}
$$

Answer $= x^2 - x - 6$

(b)

$$
\begin{array}{r}
x^2 + 3x - 4 \\
x + 3 \,\big)\overline{x^3 + 6x^2 + 5x - 12} \\
\underline{-x^3 - 3x^2 \qquad\qquad} \\
+ 3x^2 + 5x \qquad \\
\underline{- 3x^2 - 9x \qquad} \\
- 4x - 12 \\
\underline{+ 4x + 12} \\
0
\end{array}
$$

Answer $= x^2 + 3x - 4$

(c)

$$
\begin{array}{r}
2x^2 + 2x - 1 \\
x - 1 \,\big)\overline{2x^3 + 0x^2 - 3x + 1} \\
\underline{-2x^3 + 2x^2 \qquad\qquad} \\
+ 2x^2 - 3x \qquad \\
\underline{- 2x^2 + 2x \qquad} \\
- x + 1 \\
\underline{+ x - 1} \\
0
\end{array}
$$

Answer $= 2x^2 + 2x - 1$

(d)

$$
\begin{array}{r}
x^2 + 2x + 4 \\
x - 2 \,\big)\overline{x^3 + 0x^2 + 0x - 8} \\
\underline{-x^3 + 2x^2 \qquad\qquad} \\
+ 2x^2 + 0x \qquad \\
\underline{- 2x^2 + 4x \qquad} \\
+ 4x - 8 \\
\underline{- 4x + 8} \\
0
\end{array}
$$

Answer $= x^2 + 2x + 4$

Q7 (a) Perimeter $= 2\left(\dfrac{x+3}{2}\right) + 2\left(\dfrac{2x+5}{3}\right)$ units

(b) Perimeter $= (x+3) + \dfrac{(4x+10)}{3}$

$$= \frac{3(x+3) + 4x + 10}{3} = \frac{3x + 9 + 4x + 10}{3}$$

$$= \frac{7x+19}{3} \text{ units}$$

(c) Perimeter $= \dfrac{7x+19}{3} = \dfrac{7(2)+19}{3} = \dfrac{14+19}{3} = \dfrac{33}{3} = 11$ units

Q8

$$
\begin{array}{r}
x - 9 \\
x + 3 \,\big)\overline{x^2 - 6x - 27} \\
\underline{-x^2 - 3x \qquad} \\
- 9x - 27 \\
\underline{+ 9x + 27} \\
0
\end{array}
$$

Width $= x - 9$ units

Chapter 7 – Algebra 4: Linear Equations

Q1 (a) $-3 + x = 7$ Add 3 to both sides.

$\Rightarrow x = 7 + 3$

$\Rightarrow x = 10$

(b) $a + 9 = 17$ Subtract 9 from both sides.

$\Rightarrow a = 17 - 9$

$\Rightarrow a = 8$

(c) $6 = -b + 3$ Add b to both sides.

$\Rightarrow b + 6 = 3$ Subtract 6 from both sides.

$\Rightarrow b = 3 - 6$

$\Rightarrow b = -3$

(d) $-2 = k - 1$ Add 1 to both sides.

$\Rightarrow -2 + 1 = k$

$\Rightarrow -1 = k$ Note: writing $-1 = k$ is the same as writing $k = -1$.

$\Rightarrow k = -1$

(e) $14f = 182$

$\Rightarrow f = \frac{182}{14}$ Divide both sides by 14.

$\Rightarrow f = 13$

(f) $7h = -56$ Divide both sides by 7.

$\Rightarrow h = \frac{-56}{7}$

$\Rightarrow h = -8$

(g) $\frac{2y}{-3} = 4$ Multiply both sides by -3.

$\Rightarrow 2y = 4(-3)$

$\Rightarrow 2y = -12$ Divide both sides by 2.

$\Rightarrow y = \frac{-12}{2}$

$\Rightarrow y = -6$

(h) $\frac{3d}{-7} = -9$ Multiply both sides by -7.

$\Rightarrow 3d = -7(-9)$

$\Rightarrow 3d = 63$ Divide both sides by 3.

$\Rightarrow d = \frac{63}{3}$

$\Rightarrow d = 21$

Q2 (a) $12 + 5x = 47$ Subtract 12 from both sides.

$\Rightarrow 5x = 47 - 12$

$\Rightarrow 5x = 35$ Divide both sides by 5.

$\Rightarrow x = \frac{35}{5}$

$\Rightarrow x = 7$

(b) $2a - 7 = -9$ Add 7 to both sides.

$\Rightarrow 2a = -9 + 7$

$\Rightarrow 2a = -2$ Divide both sides by 2.

$\Rightarrow a = \frac{-2}{2}$

$\Rightarrow a = -1$

(c) $-3 - 8c = -11$ Add 3 to both sides.

$\Rightarrow -8c = -11 + 3$

$\Rightarrow -8c = -8$ Divide both sides by -8.

$\Rightarrow c = \frac{-8}{-8}$

$\Rightarrow c = 1$

(d) $-22 = 3y - 1$ Add 1 to both sides.

$\Rightarrow -22 + 1 = 3y$

$\Rightarrow -21 = 3y$ Divide both sides by 3.

$\Rightarrow \frac{-21}{3} = y$

$\Rightarrow -7 = y$

$\Rightarrow y = -7$

Q3 **(a)** $-2a - 14 = 5 - 3a$ Add $3a$ to both sides.

$\Rightarrow -2a - 14 + 3a = 5$

$\Rightarrow a - 14 = 5$ Add 14 to both sides.

$\Rightarrow a = 5 + 14$

$\Rightarrow a = 19$

(b) $7b - 11 = 13b + 25$ Subtract $13b$ from both sides.

$\Rightarrow 7b - 13b - 11 = 25$

$\Rightarrow -6b - 11 = 25$ Add 11 to both sides.

$\Rightarrow -6b = 25 + 11$

$\Rightarrow -6b = 36$ Divide both sides by -6.

$\Rightarrow b = \frac{36}{-6}$

$\Rightarrow b = -6$

(c) $3x - 7 = -4 + 5x$ Subtract $5x$ from both sides.

$\Rightarrow 3x - 7 - 5x = -4$

$\Rightarrow -2x - 7 = -4$ Add 7 to both sides.

$\Rightarrow -2x = -4 + 7$

$\Rightarrow -2x = 3$ Divide both sides by -2.

$\Rightarrow x = \frac{3}{-2}$

$\Rightarrow x = -\frac{3}{2}$

(d) $5 + 6d - 8 = 2d - 3 - 5d$ Collect like terms on both sides.

$\Rightarrow 6d - 3 = -3d - 3$ Add $3d$ to both sides.

$\Rightarrow 6d + 3d - 3 = -3$ Simplify and add 3 to both sides.

$\Rightarrow 9d = -3 + 3$

$\Rightarrow 9d = 0$ Divide by 9.

$\Rightarrow d = \frac{0}{9}$ Note: $\frac{0}{\text{any number}} = 0$.

$\Rightarrow d = 0$

Q4 **(a)** $6y - 3(y + 2) = 3 - 6y$ Expand the brackets.

$\Rightarrow 6y - 3y - 6 = 3 - 6y$ Collect like terms on both sides.

$\Rightarrow 3y - 6 = 3 - 6y$ Add $6y$ to both sides.

$\Rightarrow 3y + 6y - 6 = 3$ Add 6 to both sides.

$\Rightarrow 9y = 3 + 6$

$\Rightarrow 9y = 9$ Divide both sides by 9.

$\Rightarrow y = 1$

(b)	$2(c+1)-(3-2c)=2c-7(1-2c)$	Expand the brackets.
	$\Rightarrow 2c+2-3+2c=2c-7+14c$	Collect like terms on both sides.
	$\Rightarrow 4c-1=16c-7$	Subtract $16c$ from both sides.
	$\Rightarrow 4c-1-16c=-7$	Add 1 to both sides.
	$\Rightarrow -12c=-7+1$	Simplify and divide by −12.
	$\Rightarrow c=\dfrac{-6}{-12}$	
	$\Rightarrow c=\dfrac{1}{2}$	
Q5 (a)	$3-\dfrac{y-2}{4}=-4$	Multiply by 4 (the LCD). Put brackets around $y-2$.
	$\Rightarrow 12-(y-2)=-16$	Expand the brackets.
	$\Rightarrow 12-y+2=-16$	
	$\Rightarrow 14-y=-16$	Subtract 14 from both sides.
	$\Rightarrow -y=-16-14$	Remember that $-y=-1y$.
	$\Rightarrow -y=-30$	Divide both sides by −1.
	$\Rightarrow y=\dfrac{-30}{-1}$	
	$\Rightarrow y=30$	
(b)	$\dfrac{b}{5}-\dfrac{b}{2}=3$	Multiply both sides by 10 (the LCD of 2 and 5).
	$\Rightarrow 2b-5b=30$	
	$\Rightarrow -3b=30$	Divide both sides by −3.
	$\Rightarrow b=\dfrac{30}{-3}$	
	$\Rightarrow b=-10$	
(c)	$\dfrac{2}{3}(3x-5)+2=\dfrac{1}{2}(x-1)$	Multiply both sides by 6 (the LCD of 3 and 2).
	$\Rightarrow 4(3x-5)+6(2)=3(x-1)$	Expand the brackets.
	$\Rightarrow 12x-20+12=3x-3$	Simplify.
	$\Rightarrow 12x-8=3x-3$	Subtract $3x$ from both sides.
	$\Rightarrow 12x-3x-8=-3$	Simplify.
	$\Rightarrow 9x-8=-3$	Add 8 to both sides.
	$\Rightarrow 9x=-3+8$	
	$\Rightarrow 9x=5$	Divide both sides by 9.
	$\Rightarrow x=\dfrac{5}{9}$	

Q6 (a) $\frac{x+3}{4} - \frac{x-4}{2} = 7$

Multiply both sides by 4 (the LCD of 2 and 4). Put brackets around $x + 3$ and $x - 4$.

$\Rightarrow (x+3) - 2(x-4) = 4(7)$ — Expand the brackets.

$\Rightarrow x + 3 - 2x + 8 = 28$

$\Rightarrow -x + 11 = 28$

$\Rightarrow -x = 28 - 11$ — Subtract 11 from both sides.

$\Rightarrow -x = 17$ — Remember that $-x = -1x$.

$\Rightarrow x = -17$ — Divide both sides by -1.

(b) $\frac{2n-3}{2} + \frac{n+1}{3} = \frac{3n-1}{3}$

Multiply both sides by 6 (the LCD of 2 and 3). Put brackets around each numerator.

$\Rightarrow 3(2n-3) + 2(n+1) = 2(3n-1)$ — Expand the brackets.

$\Rightarrow 6n - 9 + 2n + 2 = 6n - 2$

$\Rightarrow 8n - 7 = 6n - 2$ — Subtract $6n$ from both sides.

$\Rightarrow 8n - 6n - 7 = -2$ — Add 7 to both sides.

$\Rightarrow 2n = -2 + 7$ — Divide both sides by 2.

$\Rightarrow n = \frac{5}{2}$

(c) $\frac{x}{3} + \frac{x+2}{4} = \frac{3}{4} + \frac{x}{2}$

Multiply both sides by 12 (the LCD of 2, 3 and 4). Put brackets around each numerator.

$\Rightarrow 4x + 3(x+2) = 3(3) + 6(x)$ — Expand the brackets.

$\Rightarrow 4x + 3x + 6 = 9 + 6x$ — Collect like terms on each side.

$\Rightarrow 7x + 6 = 9 + 6x$ — Subtract $6x$ from each side.

$\Rightarrow 7x - 6x + 6 = 9$ — Subtract 6 from each side.

$\Rightarrow x = 9 - 6$

$\Rightarrow x = 3$

Q7 (a) Let x = number.

$3x - 4 = 17$ — Add 4 to both sides.

$\Rightarrow 3x = 17 + 4$ — Simplify and divide by 3.

$\Rightarrow x = \frac{21}{3}$

$\Rightarrow x = 7$

The number is 7.

(b) We know:

Consecutive natural numbers are natural numbers which are next to each other (one after the other).

Let:

x = first natural number

$x + 1$ = second natural number

$x + 2$ = third natural number

First natural number + second natural number + third natural number = 99

$x + (x + 1) + (x + 2) = 99$ Expand the brackets and simplify.

$\Rightarrow 3x + 3 = 99$ Subtract 3 from both sides.

$\Rightarrow 3x = 99 - 3$

$\Rightarrow 3x = 96$ Divide both sides by 3.

$\Rightarrow x = \frac{96}{3}$

$\Rightarrow x = 32$

$x = 32$ is the first natural number.

$x + 1 = 32 + 1 = 33$ is the second natural number.

$x + 2 = 32 + 2 = 34$ is the third natural number.

Verify:

$x + (x + 1) + (x + 2) = 99$

$32 + 33 + 34 = 99$

$99 = 99$ ✓

(c) Let x = number.

$\frac{x}{4} - 6 = 15$ Add 6 to both sides.

$\Rightarrow \frac{x}{4} = 15 + 6$ Simplify.

$\Rightarrow \frac{x}{4} = 21$ Multiply both sides by 4.

$\Rightarrow x = 4(21)$

$\Rightarrow x = 84$

The number is 84.

Q8 **(a)** Let x = the number of weeks it will take for both Katie and Conor to have the same amount in their accounts.

$438 + 12x = 620 - 14x$

(b) $438 + 12x = 620 - 14x$ Add $14x$ to both sides.

$\Rightarrow 438 + 12x + 14x = 620$ Subtract 438 from both sides.

$\Rightarrow 26x = 620 - 438$

$\Rightarrow 26x = 182$ Divide both sides by 26.

$\Rightarrow x = \dfrac{182}{26}$

$\Rightarrow x = 7$

It will be 7 weeks before Katie and Conor have the same amount in their accounts.

(c) Verify:

$438 + 12x = 620 - 14x$

$438 + 12(7) = 620 - 14(7)$

$438 + 84 = 620 - 98$

$522 = 522$ ✓

Chapter 8 – Algebra 5: Solving Linear Inequalities

Q1 (a) $-2 < x < 4, x \in \mathbb{Z}$; or $-1 \leq x \leq 3, x \in \mathbb{Z}$.

(b) $0 < x < 4, x \in \mathbb{Z}$; $1 \leq x \leq 3, x \in \mathbb{N}$; or $x < 4, x \in \mathbb{N}$.

(c) $-4 \leq x < 5, x \in \mathbb{R}$.

(d) $x > -9, x \in \mathbb{R}$.

Q2 (a) $9 - 4x > -3, x \in \mathbb{N}$ Subtract 9 from each side.

$\Rightarrow 9 - 4x - 9 > -3 - 9$

$\Rightarrow -4x > -12$ Divide by −4 and change the direction of the inequality sign.

$\Rightarrow \dfrac{-4x}{-4} < \dfrac{-12}{-4}$

$\Rightarrow x < 3, x \in \mathbb{N}$

(b) $\dfrac{1 - 3x}{2} < 5, x \in \mathbb{R}$ Multiply both sides of the inequality by 2.

$\Rightarrow 2\left(\dfrac{1 - 3x}{2}\right) < 2(5)$

$\Rightarrow 1 - 3x < 10$ Subtract 1 from each side of the inequality.

$\Rightarrow 1 - 3x - 1 < 10 - 1$ Divide by −3 and change the direction of the inequality sign.

$\Rightarrow \dfrac{-3x}{-3} > \dfrac{9}{-3}$

$\Rightarrow x > -3, x \in \mathbb{R}$

Q3 (a) $14x + 1 > 2(6x - 3) - 5, x \in \mathbb{R}$ Expand the brackets.

$\Rightarrow 14x + 1 > 12x - 6 - 5$ Subtract $12x$ from both sides.

$\Rightarrow 14x - 12x + 1 > 12x - 11 - 12x$

$\Rightarrow 2x + 1 > -11$ Subtract 1 from both sides.

$\Rightarrow 2x + 1 - 1 > -11 - 1$ Divide both sides by 2.

$\Rightarrow \dfrac{2x}{2} > \dfrac{-12}{2}$

$\Rightarrow x > -6, x \in \mathbb{R}$

(b) $6x - 3 > 2(4x - 3) - 6, x \in \mathbb{R}$ Expand the brackets.

$\Rightarrow 6x - 3 > 8x - 6 - 6$ Subtract $8x$ from both sides.

$\Rightarrow 6x - 8x - 3 > 8x - 12 - 8x$

$\Rightarrow -2x - 3 > -12$ Add 3 to both sides.

$\Rightarrow -2x - 3 + 3 > -12 + 3$ Divide by -2 and change the direction of the inequality sign.

$\Rightarrow \dfrac{-2x}{-2} < \dfrac{-9}{-2}$

$\Rightarrow x < 4{\cdot}5, x \in \mathbb{R}$

Q4 (a) $5x - 3(2x - 1) \geq 2(x - 3), x \in \mathbb{N}$ Expand the brackets.

$\Rightarrow 5x - 6x + 3 \geq 2x - 6$

$\Rightarrow -x + 3 \geq 2x - 6$ Subtract $2x$ from both sides.

$\Rightarrow -x + 3 - 2x \geq 2x - 6 - 2x$

$\Rightarrow -3x + 3 \geq -6$ Subtract 3 from both sides.

$\Rightarrow -3x + 3 - 3 \geq -6 - 3$ Divide by -3 and change the direction of the inequality sign.

$\Rightarrow \dfrac{-3x}{-3} \leq \dfrac{-9}{-3}$

$\Rightarrow x \leq 3, x \in \mathbb{N}$

(b) $2(x-2) \leq 2(2x-1) < 2x+6, x \in \mathbb{Z}$ Expand the brackets.

$\Rightarrow 2x-4 \leq 4x-2 < 2x+6$ Add 2 to all parts.

$\Rightarrow 2x-4+2 \leq 4x-2+2 < 2x+6+2$

$\Rightarrow 2x-2 \leq 4x < 2x+8$ Subtract $2x$ from all parts.

$\Rightarrow 2x-2-2x \leq 4x-2x < 2x+8-2x$

$\Rightarrow -2 \leq 2x < 8$ Divide all parts by 2.

$\Rightarrow \frac{-2}{2} \leq \frac{2x}{2} < \frac{8}{2}$

$\Rightarrow -1 \leq x < 4, x \in \mathbb{Z}$

Q5 (a) $7x-3 > 18, x \in \mathbb{N}$ Add 3 to both sides.

$\Rightarrow 7x-3+3 > 18+3$

$\Rightarrow \quad 7x > 21$ Divide both sides by 7.

$\Rightarrow \quad \frac{7x}{7} > \frac{21}{7}$

$\Rightarrow \quad x > 3, x \in \mathbb{N}$

(b) $x = 4$

Q6 (a) $3x-8 < 7, x \in \mathbb{Z}$ Add 8 to both sides.

$\Rightarrow 3x-8+8 < 7+8$

$\Rightarrow \quad 3x < 15$ Divide both sides by 3.

$\Rightarrow \quad \frac{3x}{3} < \frac{15}{3}$

$\Rightarrow \quad x < 5, x \in \mathbb{Z}$

$A = \{x | x < 5, x \in \mathbb{Z}\}$

(b) $5-2x < 9, x \in \mathbb{Z}$ Subtract 5 from both sides.

$\Rightarrow 5-2x-5 < 9-5$

$\Rightarrow \quad -2x < 4$ Divide by −2 and change the direction of the inequality signs.

$\Rightarrow \quad \frac{-2x}{-2} > \frac{4}{-2}$

$\Rightarrow \quad x > -2, x \in \mathbb{Z}$

$B = \{x | x > -2, x \in \mathbb{Z}\}$

(c) $A \cap B = \{-1, 0, 1, 2, 3, 4\}$

Q7 Let x = score of fifth event that gives an average $\geq 8{\cdot}5$

$$\Rightarrow \frac{7{\cdot}5 + 8{\cdot}5 + 8{\cdot}2 + 9 + x}{5} \geq 8{\cdot}5$$

$$\Rightarrow \frac{33{\cdot}2 + x}{5} \geq 8{\cdot}5$$ Multiply both sides by 5.

$$\Rightarrow 5\left(\frac{33{\cdot}2 + x}{5}\right) \geq 5(8{\cdot}5)$$

$$\Rightarrow 33{\cdot}2 + x \geq 42{\cdot}5$$ Subtract 33·2 from both sides.

$$\Rightarrow 33{\cdot}2 + x - 33{\cdot}2 \geq 42{\cdot}5 - 33{\cdot}2$$

$$\Rightarrow x \geq 9{\cdot}3$$

The gymnast must get a minimum score of 9·3 in the fifth event to get an average of at least 8·5 overall.

Q8 **(a)** $3000 - 32x \geq 800$

(b) $3000 - 32x \geq 800$ Subtract 3000 from both sides.

$$\Rightarrow -32x \geq 800 - 3000$$

$$\Rightarrow -32x \geq -2200$$ Divide across by −32 and change the direction of the inequality sign.

$$\Rightarrow \frac{-32x}{-32} \leq \frac{-2200}{-32}$$

$$\Rightarrow x \leq 68{\cdot}75$$

Hence, the maximum amount Josephine can spend each week is €68·75.

Chapter 9 – Algebra 6: Solving Simultaneous Equations

Q1 **(a)**

$$x + 2y = 20 \quad (1)$$

$$+\ \ 9x - 2y = 80 \quad (2)$$

$$10x = 100$$ Add equation (1) and equation (2).

$$\Rightarrow x = \frac{100}{10}$$

$$\Rightarrow x = 10$$

Substitute $x = 10$ into (1) to find y.

$$x + 2y = 20 \quad (1)$$

$$\Rightarrow 10 + 2y = 20$$

$$\Rightarrow 2y = 10$$

$$\Rightarrow y = \frac{10}{2}$$

$$\Rightarrow y = 5$$

Check solution with equation (2):

$9x - 2y = 80 \Rightarrow 9(10) - 2(5) = 80$

$\Rightarrow 90 - 10 = 80 \Rightarrow 80 = 80$ ✓

(b) $4x - 3y = 3$ (1) Label the equations (1) and (2).

$2x + 6y = -1$ (2)

$$\begin{array}{rll} 8x - 6y = 6 & (1) \times 2 & \text{Eliminate the } y \text{ variable.} \\ + \underline{2x + 6y = -1} & (2) & \\ 10x = 5 & & \end{array}$$

$\Rightarrow x = \dfrac{5}{10}$

$\Rightarrow x = \dfrac{1}{2}$

Substitute $x = \dfrac{1}{2}$ into (1) to find y:

$4x - 3y = 3$ (1)

$\Rightarrow 4\left(\dfrac{1}{2}\right) - 3y = 3$

$\Rightarrow -3y = 3 - 2$

$\Rightarrow y = -\dfrac{1}{3}$

Check solution with equation (2):

$2x + 6y = -1 \Rightarrow 2\left(\dfrac{1}{2}\right) + 6\left(-\dfrac{1}{3}\right) = -1$

$\Rightarrow 1 - 2 = -1 \Rightarrow -1 = -1$ ✓

(c) $3x + 4y = -1$

$2x + 9 = -6y$

Rearrange both equations in the form $ax + by = c$ and label them (1) and (2):

$3x + 4y = -1$ (1)

$2x + 6y = -9$ (2)

Eliminate the x variable:

$$\begin{array}{rl} -6x - 8y = 2 & (1) \times -2 \\ + \underline{6x + 18y = -27} & (2) \times 3 \\ 10y = -25 & \end{array}$$

$\Rightarrow y = \dfrac{-25}{10}$

$\Rightarrow y = -2{\cdot}5$

Substitute $y = -2{\cdot}5$ into (1) to find x:

$$3x + 4y = -1 \qquad (1)$$
$$\Rightarrow \quad 3x + 4(-2{\cdot}5) = -1$$
$$\Rightarrow \quad 3x = -1 + 10$$
$$\Rightarrow \quad x = \frac{9}{3}$$
$$\Rightarrow \quad x = 3$$

Check solution with equation (2): $2x + 6y = -9 \Rightarrow 2(3) + 6(-2{\cdot}5) = -9$
$\Rightarrow 6 - 15 = -9 \Rightarrow -9 = -9$ ✓

Q2 (a) Label the linear equations (1) and (2):

$x - 2y = 5 \qquad (1)$

$x + 5y = -9 \qquad (2)$

Rearrange equation (2) in the form '$x =$':

$x = -5y - 9$

Substitute $x = -5y - 9$ into equation (1) to find y:

$$x - 2y = 5 \qquad (1)$$
$$\Rightarrow \quad (-5y - 9) - 2y = 5$$
$$\Rightarrow \quad -7y = 14$$
$$\Rightarrow \quad y = \frac{14}{-7}$$
$$\Rightarrow \quad y = -2$$

Substitute $y = -2$ into $x = -5y - 9$:

$$x = -5(-2) - 9$$
$$\Rightarrow \quad x = 10 - 9$$
$$\Rightarrow \quad x = 1$$

Check solution with equation (2): $x + 5y = -9 \Rightarrow (1) + 5(-2) = -9$

$\Rightarrow 1 - 10 = -9 \Rightarrow -9 = -9$ ✓

(b) Label the linear equations as (1) and (2):

$3x + 4y = 1 \qquad (1)$

$x = -3 + 2y \qquad (2)$

Substitute $-3 + 2y$ into equation (1) for x:

$$3x + 4y = 1 \qquad (1)$$
$$\Rightarrow \quad 3(-3 + 2y) + 4y = 1$$
$$\Rightarrow \quad -9 + 6y + 4y = 1$$

$\Rightarrow \quad 10y = 1 + 9$

$\Rightarrow \quad y = \frac{10}{10}$

$\Rightarrow \quad y = 1$

Substitute $y = 1$ into $x = -3 + 2y$ to find x:

$x = -3 + 2(1)$

$\Rightarrow \quad x = -3 + 2$

$\Rightarrow \quad x = -1$

Check solution: $3x + 4y = 1 \Rightarrow 3(-1) + 4(1) = 1$

$\Rightarrow -3 + 4 = 1 \Rightarrow 1 = 1$ ✓

(c) Label the linear equations as (1) and (2):

$a - 2b = -3 \qquad (1)$

$a + 2b = 1 \qquad (2)$

Rearrange equation (2) in the form '$a =$':

$a = -2b + 1$

Substitute $-2b + 1$ into equation (1) for a:

$a - 2b = -3 \qquad (1)$

$\Rightarrow \quad (-2b + 1) - 2b = -3$

$\Rightarrow \quad -2b + 1 - 2b = -3$

$\Rightarrow \quad -4b = -4$

$\Rightarrow \quad b = \frac{-4}{-4}$

$\Rightarrow \quad b = 1$

Substitute $b = 1$ into $a = -2b + 1$ to find a:

$a = -2b + 1$

$\Rightarrow \quad a = -2(1) + 1$

$\Rightarrow \quad a = -2 + 1$

$\Rightarrow \quad a = -1$

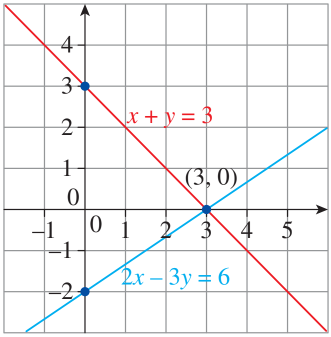
Check solution: $a - 2b = -3 \Rightarrow -1 - 2(1) = -3$

$\Rightarrow -1 - 2 = -3 \Rightarrow -3 = -3$ ✓

Q3 (a) Label the equations (1) and (2).

$x + y = 3$ (1)

$2x - 3y = 6$ (2)



Find two points for each linear equation.

Equation (1):

$x + y = 3$ (1)

When $x = 0$, find y:

$0 + y = 3 \Rightarrow y = 3$ Point 1 is (0, 3).

When $y = 0$, find x:

$x + 0 = 3 \Rightarrow x = 3$ Point 2 is (3, 0).

Equation (2):

$2x - 3y = 6$ (2)

When $x = 0$, find y:

$0 - 3y = 6 \Rightarrow y = \frac{6}{-3} = -2$ Point 3 is (0, −2).

When $y = 0$, find x:

$2x - 0 = 6 \Rightarrow x = \frac{6}{2} = 3$ Point 4 is (3, 0).

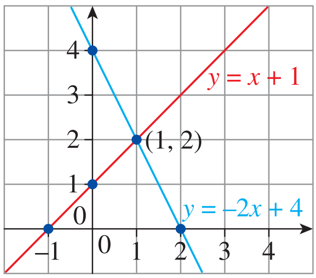
From the graph the point of intersection is (3, 0).

Solution: $x = 3$ and $y = 0$

(b) Label the equations (1) and (2).

$y = x + 1$ (1)

$y = -2x + 4$ (2)



Find two points for each linear equation.

Equation (1):

$y = x + 1$ (1)

When $x = 0$, find y:

$y = 0 + 1 \Rightarrow y = 1$ Point 1 is (0, 1).

When $y = 0$, find x:

$0 = x + 1 \Rightarrow x = -1$ Point 2 is (−1, 0).

Equation (2):

$y = -2x + 4$ (2)

When $x = 0$, find y:

$y = 0 + 4 \Rightarrow y = 4$ Point 3 is (0, 4).

When $y = 0$, find x:

$0 = -2x + 4 \Rightarrow 2x = 4 \Rightarrow x = 2$ Point 4 is (2, 0).

From the graph, the point where the two lines cross / the point of intersection is (1, 2).

Solution: $x = 1$ and $y = 2$.

(c) Label the equations (1) and (2).

$x - 2y = 5$ (1)

$x + 5y = -9$ (2)

Find two points for each linear equation.

Equation (1):

$x - 2y = 5$ (1)

When $x = 0$, find y:

$0 - 2y = 5 \Rightarrow y = -2{\cdot}5$ Point 1 is (0, −2·5).

When $y = 0$, find x:

$x + 0 = 5 \Rightarrow x = 5$ Point 2 is (5, 0).

Equation (2):

$x + 5y = -9$ (2)

When $x = 0$, find y:

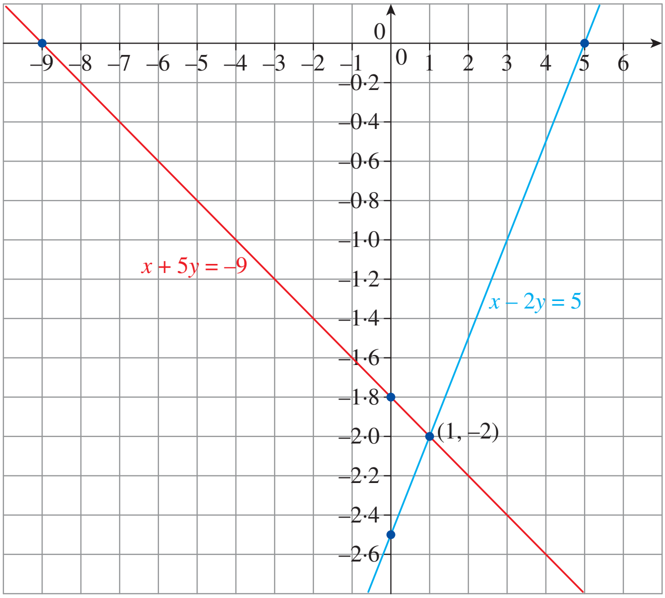
$0 + 5y = -9$

$\Rightarrow 5y = -9 \Rightarrow y = -1{\cdot}8$ Point 3 is (0, −1.8).

When $y = 0$, find x:

$x + 5(0) = -9 \Rightarrow x = -9$ Point 4 is (−9, 0).

From the graph, the point where the two lines cross / the point of intersection is (1, −2).



Solution: $x = 1$ and $y = -2$.

Q4 (a) Rearrange $\frac{x}{3} - y = -1$ in the form $ax + by = c$, where $a, b, c \in \mathbb{Z}$:

$$\frac{x}{3} - y = -1 \qquad \text{Multiply by 3.}$$

$\Rightarrow \quad x - 3y = -3$

Label the equations (1) and (2):

$x - 3y = -3 \qquad (1)$

$4x - y = 10 \qquad (2)$

Eliminate the y variable:

$$\begin{array}{rrl} & x - 3y = -3 & (1) \\ + & -12x + 3y = -30 & (2) \times -3 \\ \hline & -11x = -33 & \end{array}$$

$\Rightarrow \quad x = \frac{-33}{-11}$

$\Rightarrow \quad x = 3$

Substitute $x = 3$ into (2) to find y:

$4x - y = 10$

$\Rightarrow \quad 4(3) - 10 = y$

$\Rightarrow \quad y = 12 - 10$

$\Rightarrow \quad y = 2$

Check your solution by substituting both values into equation (1):

$x - 3y = -3 \Rightarrow 3 - 3(2) = -3$

$\Rightarrow 3 - 6 = -3 \Rightarrow -3 = -3 \quad \checkmark$

(b) Rearrange both equations in the form $ax + by = c$, where $a, b, c \in \mathbb{Z}$:

$\frac{1}{2}x - y = -5$ Multiply by 2.

$\Rightarrow x - 2y = -10$

$\frac{5}{2}x + y = -1$ Multiply by 2.

$\Rightarrow 5x + 2y = -2$

Label the equations (1) and (2) and eliminate the y variable:

$$\begin{array}{rll} & x - 2y = -10 & (1) \\ + & 5x + 2y = -2 & (2) \\ \hline & 6x = -12 & \end{array}$$

$\Rightarrow x = \frac{-12}{6}$

$\Rightarrow x = -2$

Substitute $x = -2$ into (1) to find y:

$x - 2y = -10$

$\Rightarrow -2 - 2y = -10$

$\Rightarrow -2y = -10 + 2$

$\Rightarrow y = \frac{-8}{-2}$

$\Rightarrow y = 4$

Check your solution by substituting both values into equation (2):

$5x + 2y = -2 \Rightarrow 5(-2) + 2(4) = -2$

$\Rightarrow -10 + 8 = -2 \Rightarrow -2 = -2$ ✓

(c) Rearrange both equations in the form $ax + by = c$, where $a, b, c \in \mathbb{Z}$:

$\frac{2x}{5} - \frac{2y}{3} = \frac{12}{5}$ Multiply by the LCD, 15.

$\Rightarrow 15\left(\frac{2x}{5}\right) - 15\left(\frac{2y}{3}\right) = 15\left(\frac{12}{5}\right)$

$\Rightarrow 3(2x) - 5(2y) = 3(12)$

$\Rightarrow 6x - 10y = 36$

$\frac{x-3}{4} - \frac{y-1}{3} = \frac{5}{6}$ Multiply by the LCD, 12.

$\Rightarrow 12\left(\frac{x-3}{4}\right) - 12\left(\frac{y-1}{3}\right) = 12\left(\frac{5}{6}\right)$

$\Rightarrow 3(x-3) - 4(y-1) = 2(5)$

$\Rightarrow 3x - 9 - 4y + 4 = 10$

$\Rightarrow 3x - 4y = 15$

Label the equations (1) and (2):

$6x - 10y = 36 \qquad (1)$

$3x - 4y = 15 \qquad (2)$

Eliminate the x variable:

$$\begin{array}{rll} & 6x - 10y = 36 & (1) \\ + & \underline{-6x + 8y = -30} & (2) \times -2 \\ & -2y = 6 & \end{array}$$

$\Rightarrow \quad y = \dfrac{6}{-2}$

$\Rightarrow \quad y = -3$

Substitute $y = -3$ into (2) to find x:

$3x - 4y = 15 \qquad (2)$

$\Rightarrow \quad 3x - 4(-3) = 15$

$\Rightarrow \quad 3x = 15 - 12$

$\Rightarrow \quad x = \dfrac{3}{3}$

$\Rightarrow \quad x = 1$

Check your solution by substituting both values into equation (1):

$6x - 10y = 36 \Rightarrow 6(1) - 10(-3) = 36$

$\Rightarrow 6 + 30 = 36 \Rightarrow 36 = 36$ ✓

Q5 (a) Write two linear equations to represent the information provided and label them (1) and (2):

Let x = admission price for an adult to Dublin Zoo.

Let y = admission price for a child to Dublin Zoo.

Henry family: $2x + 3y = 68{\cdot}40 \qquad (1)$

Clarke family: $3x + 4y = 96{\cdot}70 \qquad (2)$

(b) Eliminate the x variable:

$$\begin{array}{rll} & 6x + 9y = 205{\cdot}20 & (1) \times 3 \\ + & \underline{-6x - 8y = -193{\cdot}40} & (2) \times -2 \\ & y = 11{\cdot}80 & \end{array}$$

Substitute $y = 11{\cdot}80$ into (1) to find x.

$2x + 3y = 68{\cdot}40$

$\Rightarrow \quad 2x + 3(11{\cdot}80) = 68{\cdot}40$

$\Rightarrow \quad 2x + 35{\cdot}40 = 68{\cdot}40$

$\Rightarrow \quad 2x = 68{\cdot}40 - 35{\cdot}40$

$\Rightarrow \quad 2x = 33$

$\Rightarrow \qquad x = \frac{33}{2}$

$\Rightarrow \qquad x = 16{\cdot}5$

An adult ticket costs €16·50.

A child ticket costs €11·80.

(c) Verify the solution by substituting $x = 16{\cdot}50$ and $y = 11{\cdot}80$ into (2).

$3x + 4y = 96{\cdot}70 \Rightarrow 3(16{\cdot}5) + 4(11{\cdot}80) = 96{\cdot}70$

$\Rightarrow 49{\cdot}50 + 47{\cdot}20 = 96{\cdot}70 \Rightarrow 96{\cdot}70 = 96{\cdot}70$ ✓

Q6 (a) Write two linear equations to represent the information provided and label them (1) and (2):

Let x = number of 1-cent coins

Let y = number of 2-cent coins

$x + y = 250$ (1)

$x + 2y = 410$ (2) €4·10 equals 410 cents

(b) Eliminate the x variable:

$$\begin{array}{rr} & x + y = 250 \\ + & -x - 2y = -410 \\ \hline & -y = -160 \end{array} \qquad \begin{array}{l} (1) \\ (2) \times -1 \\ \\ \end{array}$$

$\Rightarrow \qquad y = \frac{-160}{-1}$

$\Rightarrow \qquad y = 160$

Substitute $y = 160$ into (1) to find x.

$x + y = 250$

$\Rightarrow \quad x + 160 = 250$

$\Rightarrow \qquad x = 250 - 160$

$\Rightarrow \qquad x = 90$

There are ninety 1-cent coins

There are one hundred and sixty 2-cent coins.

(c) Verify the solution by substituting $x = 90$ and $y = 160$ into (2).

$x + 2y = 410 \Rightarrow 90 + 2(160) = 410 \Rightarrow 90 + 320 = 410 \Rightarrow 410 = 410$ ✓

Chapter 10 – Algebra 7: Solving Quadratic Equations

Q1 (a) Rewrite $36x^2 = 9$ as $ax^2 + bx + c = 0$

$\Rightarrow 36x^2 - 9 = 0$ Difference of two squares.

$\Rightarrow (6x)^2 - (3)^2 = 0$ Factorise.

$\Rightarrow (6x - 3)(6x + 3) = 0$

$\Rightarrow 6x = 3$ or $6x = -3$

$\Rightarrow x = \frac{1}{2}$ or $x = -\frac{1}{2}$

(b) $2x^2 - 32 = 0$ Take out HCF = 2.

$\Rightarrow 2(x^2 - 16) = 0$ Difference of two squares.

$\Rightarrow 2((x)^2 - (4)^2) = 0$ Factorise.

$\Rightarrow 2(x - 4)(x + 4) = 0$ Divide by 2.

$\Rightarrow (x - 4)(x + 4) = 0$

$\Rightarrow x - 4 = 0$ or $x + 4 = 0$

$\Rightarrow x = 4$ or $x = -4$

(c) $c^2 - 2c + 1 = 0$ Factorise the LHS.

$\Rightarrow (c - 1)(c - 1) = 0$

$\Rightarrow c - 1 = 0$

$\Rightarrow c = 1$ Only one root means the curve touches the x-axis only once.

(d) $x^2 + 11x - 12 = 0$ Factorise the LHS.

$\Rightarrow (x - 1)(x + 12) = 0$

$\Rightarrow x - 1 = 0$ or $x + 12 = 0$

$\Rightarrow x = 1$ or $x = -12$

Q2 (a) $2x^2 - 5x - 3 = 0$ Guide number = −6. Factors which add to $-5x$ are $(+x)(-6x)$.

$\Rightarrow 2x^2 + x - 6x - 3 = 0$ Factorise.

$\Rightarrow x(2x + 1) - 3(2x + 1) = 0$

$\Rightarrow (x - 3)(2x + 1) = 0$

$\Rightarrow x - 3 = 0$ or $2x + 1 = 0$

$\Rightarrow x = 3$ or $x = -\frac{1}{2}$

(b) $22x^2 - 13x + 1 = 0$

Guide number = +22. Factors which add to $-13x$ are $(-11x)(-2x)$.

$\Rightarrow 22x^2 - 11x - 2x + 1 = 0$

Factorise.

$\Rightarrow 11x(2x - 1) - 1(2x - 1) = 0$

$\Rightarrow (11x - 1)(2x - 1) = 0$

$\Rightarrow 11x - 1 = 0 \quad \text{or} \quad 2x - 1 = 0$

$\Rightarrow x = \dfrac{1}{11} \quad \text{or} \quad x = \dfrac{1}{2}$

(c) $7x^2 + 19x - 6 = 0$

Guide number = −42. Factors which add to $19x$ are $(21x)(-2x)$.

$\Rightarrow 7x^2 + 21x - 2x - 6 = 0$

Factorise.

$\Rightarrow 7x(x + 3) - 2(x + 3) = 0$

$\Rightarrow (7x - 2)(x + 3) = 0$

$\Rightarrow 7x - 2 = 0 \quad \text{or} \quad x + 3 = 0$

$\Rightarrow x = \dfrac{2}{7} \quad \text{or} \quad x = -3$

(d) $-4x^2 - 7x + 2 = 0$

Guide number = −8. Factors which add to $-7x$ are $(-8x)(+1x)$.

$\Rightarrow -4x^2 - 8x + 1x + 2 = 0$

Factorise.

$\Rightarrow -4x(x + 2) + 1(x + 2) = 0$

$\Rightarrow (-4x + 1)(x + 2) = 0$

$\Rightarrow -4x + 1 = 0 \quad \text{or} \quad x + 2 = 0$

$\Rightarrow x = \dfrac{1}{4} \quad \text{or} \quad x = -2$

Q3 (a) $x^2 - 7x + 2 = 0$

We know that: $a = 1$, $b = -7$ and $c = 2$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substitute $a = 1$, $b = -7$ and $c = 2$ into the quadratic formula.

$$\Rightarrow x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(2)}}{2(1)}$$

$$\Rightarrow x = \frac{7 \pm \sqrt{49 - 8}}{2}$$

$$\Rightarrow x = \frac{7 + \sqrt{41}}{2} \quad \text{or} \quad x = \frac{7 - \sqrt{41}}{2}$$

These are the roots in surd form.

(b) $3x^2 - 4x - 5 = 0$

We know that: $a = 3$, $b = -4$ and $c = -5$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substitute $a = 3$, $b = -4$ and $c = -5$ into the quadratic formula.

$$\Rightarrow x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-5)}}{2(3)}$$

$$\Rightarrow x = \frac{4 \pm \sqrt{16 + 60}}{6}$$

$$\Rightarrow x = \frac{4 \pm \sqrt{76}}{6}$$

$$\Rightarrow x = \frac{4 \pm 2\sqrt{19}}{6}$$

$$\Rightarrow x = \frac{2 + \sqrt{19}}{3} \quad \text{or} \quad x = \frac{2 - \sqrt{19}}{3}$$

These are the roots in surd form.

Q4 (a) $2x^2 + 9x - 13 = 0$

We know that: $a = 2$, $b = 9$ and $c = -13$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substitute $a = 2$, $b = 9$ and $c = -13$ into the quadratic formula.

$$\Rightarrow x = \frac{-(9) \pm \sqrt{(9)^2 - 4(2)(-13)}}{2(2)}$$

$$\Rightarrow x = \frac{-9 \pm \sqrt{81 + 104}}{4}$$

$$\Rightarrow x = \frac{-9 \pm \sqrt{185}}{4}$$

$$\Rightarrow x = \frac{-9 + \sqrt{185}}{4} \quad \text{or} \quad x = \frac{-9 - \sqrt{185}}{4}$$

Use a calculator to solve correct to two decimal places.

$$\Rightarrow x = 1{\cdot}15 \quad \text{or} \quad x = -5{\cdot}65$$

(b) $8x^2 + 7x - 8 = 0$

We know that: $a = 8$, $b = 7$ and $c = -8$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substitute $a = 8$, $b = 7$ and $c = -8$ into the quadratic formula.

$$\Rightarrow x = \frac{-(7) \pm \sqrt{(7)^2 - 4(8)(-8)}}{2(8)}$$

$$\Rightarrow x = \frac{-7 \pm \sqrt{49 + 256}}{16}$$

$$\Rightarrow x = \frac{-7 \pm \sqrt{305}}{16}$$

$$\Rightarrow x = \frac{-7 + \sqrt{305}}{16} \quad \text{or} \quad x = \frac{-7 - \sqrt{305}}{16}$$

Use a calculator to solve correct to two decimal places.

$$\Rightarrow x = 0{\cdot}65 \quad \text{or} \quad x = -1{\cdot}53$$

(c) $5x^2 - 3x - 4 = 0$

We know that: $a = 5$, $b = -3$ and $c = -4$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substitute $a = 5$, $b = -3$ and $c = -4$ into the quadratic formula.

$$\Rightarrow x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(5)(-4)}}{2(5)}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{9 + 80}}{10}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{89}}{10}$$

$$\Rightarrow x = \frac{3 + \sqrt{89}}{10} \quad \text{or} \quad x = \frac{3 - \sqrt{89}}{10}$$

Use a calculator to solve correct to two decimal places.

$$\Rightarrow x = 1{\cdot}24 \quad \text{or} \quad x = -0{\cdot}64$$

(d) $6x^2 + 5x - 7 = 0$

We know that: $a = 6$, $b = 5$ and $c = -7$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substitute $a = 6$, $b = 5$ and $c = -7$ into the quadratic formula.

$$\Rightarrow x = \frac{-(5) \pm \sqrt{(5)^2 - 4(6)(-7)}}{2(6)}$$

$$\Rightarrow x = \frac{-5 \pm \sqrt{25 + 168}}{12}$$

$$\Rightarrow x = \frac{-5 \pm \sqrt{193}}{12}$$

$$\Rightarrow x = \frac{-5 + \sqrt{193}}{12} \quad \text{or} \quad x = \frac{-5 - \sqrt{193}}{12}$$

Use a calculator to solve correct to two decimal places.

$$\Rightarrow x = 0{\cdot}74 \quad \text{or} \quad x = -1{\cdot}57$$

Q5 (a) $\frac{x^2}{4}+2x-\frac{9}{4}=0$ Multiply by the LCD = 4.

$\Rightarrow x^2+8x-9=0$

$\Rightarrow x^2+9x-x-9=0$

$\Rightarrow x(x+9)-1(x+9)=0$ Factorise.

$\Rightarrow (x-1)(x+9)=0$

$\Rightarrow x-1=0$ or $x+9=0$

$\Rightarrow x=1$ or $x=-9$

(b) $\frac{x^2-3x}{2}=5$ Multiply by the LCD = 2.

$\Rightarrow x^2-3x-10=0$

$\Rightarrow x^2-5x+2x-10=0$

$\Rightarrow x(x-5)+2(x-5)=0$ Factorise.

$\Rightarrow (x+2)(x-5)=0$

$\Rightarrow x+2=0$ or $x-5=0$

$\Rightarrow x=-2$ or $x=5$

(c) $\frac{x^2}{5}-\frac{x}{2}=\frac{3}{10}$ Multiply by the LCD = 10.

$\Rightarrow 2x^2-5x-3=0$

$\Rightarrow 2x^2-6x+x-3=0$

$\Rightarrow 2x(x-3)+1(x-3)=0$ Factorise.

$\Rightarrow (2x+1)(x-3)=0$

$\Rightarrow 2x+1=0$ or $x-3=0$

$\Rightarrow x=-\frac{1}{2}$ or $x=3$

(d) $\frac{2x^2}{3}=\frac{5x}{6}+\frac{7}{2}$ Multiply by the LCD = 6.

$\Rightarrow 4x^2-5x-21=0$

$\Rightarrow 4x^2-12x+7x-21=0$

$\Rightarrow 4x(x-3)+7(x-3)=0$ Factorise.

$\Rightarrow (4x+7)(x-3)=0$

$\Rightarrow 4x+7=0$ or $x-3=0$

$\Rightarrow x=-\frac{7}{4}$ or $x=3$

Q6 (a) Verify that one of the roots is $2-\sqrt{5}$.

$x^2-4x-1=0$

We know that: $a=1$, $b=-4$ and $c=-1$.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Substitute $a=1$, $b=-4$ and $c=-1$ into the quadratic formula.

$$\Rightarrow x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-1)}}{2(1)}$$

$$\Rightarrow x=\frac{4\pm\sqrt{16+4}}{2}$$

$$\Rightarrow x=\frac{4+\sqrt{20}}{2} \quad \text{or} \quad x=\frac{4-\sqrt{20}}{2}$$

Simplify $\sqrt{20}=2\sqrt{5}$ using a calculator.

$$\Rightarrow x=\frac{4+2\sqrt{5}}{2} \quad \text{or} \quad x=\frac{4-2\sqrt{5}}{2}$$

Leave the roots in surd form.

$$\Rightarrow x=2+\sqrt{5} \quad \text{or} \quad x=2-\sqrt{5}$$

We have verified that $2-\sqrt{5}$ is a root of the equation $x^2-4x-1=0$.

(b) Verify that one of the roots is $4+\sqrt{11}$.

$x^2-8x+5=0$

We know that: $a=1$, $b=-8$ and $c=5$.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Substitute $a=1$, $b=-8$ and $c=5$ into the quadratic formula.

$$\Rightarrow x=\frac{-(-8)\pm\sqrt{(-8)^2-4(1)(5)}}{2(1)}$$

$$\Rightarrow x=\frac{8\pm\sqrt{64-20}}{2}$$

$$\Rightarrow x=\frac{8+\sqrt{44}}{2} \quad \text{or} \quad x=\frac{8-\sqrt{44}}{2}$$

Simplify $\sqrt{44}=2\sqrt{11}$ using a calculator.

$$\Rightarrow x=\frac{8+2\sqrt{11}}{2} \quad \text{or} \quad x=\frac{8-2\sqrt{11}}{2}$$

Leave the roots in surd form.

$$\Rightarrow x=4+\sqrt{11} \quad \text{or} \quad x=4-\sqrt{11}$$

We have verified that $4+\sqrt{11}$ is a root of the equation $x^2-8x+5=0$.

Q7 The roots are $x=7$ and $x=-6$

The sum of the roots $=7-6=1$

The product of the roots $=(7)(-6)=-42$

Substitute the value for the sum and the product into the quadratic formula:

$x^2 -$ (sum of the roots)$x +$ (product of the roots) $= 0$

$\Rightarrow x^2 - (1)x + (-42) = 0$

$\Rightarrow x^2 - x - 42 = 0$

Hence, $b = -1$ and $c = -42$.

Q8 (a) $2x^2 - 5x - 3 = 0$ Factorise the LHS.

$\Rightarrow 2x^2 - 6x + x - 3 = 0$

$\Rightarrow 2x(x - 3) + 1(x - 3) = 0$

$\Rightarrow (2x + 1)(x - 3) = 0$

$\Rightarrow 2x + 1 = 0$ or $x - 3 = 0$

$\Rightarrow x = -\frac{1}{2}$ or $x = 3$ These values for x are the roots.

(b) The equation $2(y - 3)^2 - 5(y - 3) - 3 = 0$ is of the same form as $2x^2 - 5x - 3 = 0$ except that x has been replaced with $(y - 3)$.

We can therefore replace x by $(y - 3)$ in the answers.

$y - 3 = x$

$\therefore y - 3 = -\frac{1}{2}$ or $y - 3 = 3$

$\therefore y = -\frac{1}{2} + 3$ or $y = 3 + 3$

$\therefore y = 2{\cdot}5$ or $y = 6$

Q9 (a) $A = x - 1,\ B = 7x + 2$

(b) $(x - 1)(7x + 2) = 2$

$\Rightarrow x(7x + 2) - 1(7x + 2) = 2$

$\Rightarrow 7x^2 + 2x - 7x - 2 = 2$

$\Rightarrow 7x^2 - 5x - 4 = 0$

(c) $7x^2 - 5x - 4 = 0$ We know that: $a = 7$, $b = -5$ and $c = -4$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substitute $a = 7$, $b = -5$ and $c = -4$ into the quadratic formula.

$$\Rightarrow x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(7)(-4)}}{2(7)}$$

$$\Rightarrow x = \frac{5 \pm \sqrt{25 + 112}}{14}$$

$$\Rightarrow x = \frac{5 \pm \sqrt{137}}{14}$$

$\Rightarrow x = \dfrac{5+\sqrt{137}}{14}$ or $x = \dfrac{5-\sqrt{137}}{14}$ Use a calculator to solve correct to two decimal places.

$\Rightarrow x = 1{\cdot}19$ or $x = -0{\cdot}48$

Q10 (a) Let x represent the number of students in her class.

Cost per calculator on website $C = \dfrac{480}{x}$.

Cost per calculator on website $D = \dfrac{480}{x+4}$.

(b) $\dfrac{480}{x} - \dfrac{480}{x+4} = 6$ Multiply across by the LCD $= x(x+4)$.

$\Rightarrow x(x+4)\dfrac{480}{x} - x(x+4)\dfrac{480}{x+4} = 6x(x+4)$

$\Rightarrow (x+4)480 - x(480) = 6x^2 + 24x$

$\Rightarrow 480x + 1920 - 480x - 6x^2 - 24x = 0$

$\Rightarrow 6x^2 + 24x - 1920 = 0$ Divide by 6.

$\Rightarrow x^2 + 4x - 320 = 0$

$\Rightarrow (x-16)(x+20) = 0$

$\Rightarrow x - 16 = 0$ or $x + 20 = 0$

$\Rightarrow x = 16$ or $x = -20$

Answer: $x = 16$, as you cannot have a negative number of calculators.

Chapter 11 – Algebra 8: Indices, Exponential Equations and Manipulating Formulae

Q1 (a) $z + 2y = 4{\cdot}641 \times 10^6 + 2(3{\cdot}241 \times 10^4)$

$= 4{\cdot}641 \times 10^6 + 6{\cdot}482 \times 10^4$

- Write the number in decimal form, then add the numbers.

  $4{\cdot}641 \times 10^6 + 6{\cdot}482 \times 10^4 = 4\,641\,000 + 64\,820$

  $= 4\,705\,820$

- Rewrite your answer in scientific notation.
  $4\,705\,820 = 4{\cdot}7 \times 10^6$, correct to two significant figures

(b) $\dfrac{2z-x}{y}=\dfrac{2(4{\cdot}641\times10^6)-(1{\cdot}234\times10^5)}{3{\cdot}241\times10^4}$

$=\dfrac{(9{\cdot}282\times10^6)-(1{\cdot}234\times10^5)}{3{\cdot}241\times10^4}$

$=\dfrac{9\,282\,000-123\,400}{32\,410}$

$=\dfrac{9\,158\,600}{32\,410}=282{\cdot}5856=2{\cdot}8\times10^2$, correct to two significant figures

Q2 Jupiter $=1{\cdot}8986\times10^{27}$ kg

Mars $=6{\cdot}4185\times10^{23}$ kg

Pluto $=1{\cdot}25\times10^{22}$ kg

Combined mass $=1{\cdot}8986\times10^{27}+6{\cdot}4185\times10^{23}+1{\cdot}25\times10^{22}$

$=1{\cdot}8986\times10^{27}+0{\cdot}000\,641\,85\times10^{27}+0{\cdot}000\,012\,5\times10^{27}$

$=1{\cdot}899\times10^{27}=1{\cdot}9\times10^{27}$ kg, correct to two significant figures

Q3 (a) $2^2 2^3=2^{2+3}=2^5=32$ — $a^p a^q=a^{p+q}$.

(b) $\dfrac{5^8}{5^3}=5^{8-3}=5^5=3125$ — $\dfrac{a^p}{a^q}=a^{p-q}$.

(c) $\dfrac{11^4}{11^4}=11^{4-4}=11^0=1$ — $\dfrac{a^p}{a^q}=a^{p-q}$ and $a^0=1$.

(d) $(3)^{-2}=\dfrac{1}{3^2}=\dfrac{1}{9}$ — Use the rule $a^{-p}=\dfrac{1}{a^p}$.

(e) $\left(\dfrac{1}{2}\right)^{-1}=\dfrac{2}{1}=2$ — Use the rule $a^{-p}=\dfrac{1}{a^p}$.

(f) $(49)^{\frac{1}{2}}=\sqrt{49}=\pm7$ — Rule $a^{\frac{1}{q}}=\sqrt[q]{a}$.

(g) $\left(\dfrac{1}{144}\right)^{\frac{1}{2}}=\sqrt{\left(\dfrac{1}{144}\right)}=\dfrac{\sqrt{1}}{\sqrt{144}}=\pm\dfrac{1}{12}$ — Use the rules $a^{\frac{1}{q}}=\sqrt[q]{a}$ and $\sqrt{\dfrac{a}{b}}=\dfrac{\sqrt{a}}{\sqrt{b}}$.

(h) $(32)^{\frac{4}{5}}=\sqrt[5]{(32)^4}=\sqrt[5]{1\,048\,576}=16$ — Use the rule $a^{\frac{p}{q}}=\sqrt[q]{a^p}$.

or $(32)^{\frac{4}{5}}=(2^5)^{\frac{4}{5}}=2^{5\times\frac{4}{5}}=2^4=16$ — Use the rule $(a^p)^q=a^{pq}$.

(i) $(2^3)^2=2^{3\times2}=2^6=64$ — Use the rule $(a^p)^q=a^{pq}$.

(j) $\left(\dfrac{5}{2}\right)^2=\dfrac{5^2}{2^2}=\dfrac{25}{4}$ — Use the rule $\left(\dfrac{a}{b}\right)^p=\dfrac{a^p}{b^p}$.

Q4 $\dfrac{(x^5)(x^2)\left(\sqrt[3]{x^2}\right)}{(x^3)\left(x^{\frac{4}{3}}\right)}$ — Use the rule $a^{\frac{p}{q}} = \sqrt[q]{a^p}$ to simplify $\sqrt[3]{x^2}$.

$= \dfrac{(x^5)(x^2)\left(x^{\frac{2}{3}}\right)}{(x^3)\left(x^{\frac{4}{3}}\right)}$ — Use the rule $a^p a^q = a^{p+q}$ to simplify the numerator and the denominator.

$= \dfrac{x^{5+2+\frac{2}{3}}}{x^{3+\frac{4}{3}}} = \dfrac{x^{7\frac{2}{3}}}{x^{4\frac{1}{3}}}$ — Use the rule $\dfrac{a^p}{a^q} = a^{p-q}$.

$= x^{7\frac{2}{3}-4\frac{1}{3}} = x^{\frac{10}{3}}$

Q5 (a) $2^{3x+12} = 16^x$ — Write 16 as a power of 2.

$\Rightarrow 2^{3x+12} = (2^4)^x$ — Rule $(a^p)^q = a^{pq}$.

$\Rightarrow 2^{3x+12} = 2^{4x}$ — Equate the indices.

$\Rightarrow 3x + 12 = 4x$

$\Rightarrow 12 = 4x - 3x$

$\Rightarrow x = 12$

(b) $25^n = 5^{2+n}$ — Write 25 as a power of 5.

$\Rightarrow (5^2)^n = (5)^{2+n}$ — Rule $(a^p)^q = a^{pq}$.

$\Rightarrow 5^{2n} = 5^{2+n}$ — Equate the indices.

$\Rightarrow 2n = 2 + n$

$\Rightarrow 2n - n = 2$

$\Rightarrow n = 2$

(c) $\dfrac{\sqrt{8}}{16} = \left(\dfrac{1}{4}\right)^{2x-1}$ — Write 8, 16 and 4 as powers of 2.

$\Rightarrow \dfrac{\sqrt{2^3}}{2^4} = \left(\dfrac{1}{2^2}\right)^{2x-1}$ — Simplify $\sqrt{2^3}$ and $\dfrac{1}{2^2}$ using $a^{\frac{p}{q}} = \sqrt[q]{a^p}$ and $a^{-p} = \dfrac{1}{a^p}$ rules.

$\Rightarrow \dfrac{2^{\frac{3}{2}}}{2^4} = (2^{-2})^{2x-1}$ — Use $\dfrac{a^p}{a^q} = a^{p-q}$ and $(a^p)^q = a^{pq}$ to simplify the LHS and RHS.

$\Rightarrow 2^{\frac{3}{2}-4} = 2^{-4x+2}$

$\Rightarrow 2^{-2\cdot5} = 2^{-4x+2}$ — Equate indices.

$\Rightarrow -2\cdot5 = -4x + 2$

$\Rightarrow 4x = 2\cdot5 + 2$

$\Rightarrow x = \dfrac{4\cdot5}{4} = \dfrac{9}{8}$

Q6 (a) $\sqrt{32}$ — Use the rule $a^{\frac{1}{q}} = \sqrt[q]{a}$.

$= (32)^{\frac{1}{2}}$ — Write 32 as a power of 2.

$= (2^5)^{\frac{1}{2}}$ — Use the rule $(a^p)^q = a^{pq}$.

$= 2^{\frac{5}{2}}$

(b) $\dfrac{2^5}{2^x} = \sqrt{32}$ Write 32 as a power of 2.

$\dfrac{2^5}{2^x} = \sqrt{2^5}$ Rules $a^{\frac{p}{q}} = \sqrt[q]{a^p}$ and $\dfrac{a^p}{a^q} = a^{p-q}$.

$\Rightarrow 2^{5-x} = 2^{\frac{5}{2}}$ Equate indices.

$\Rightarrow 5 - x = 2{\cdot}5$

$\Rightarrow x = 2{\cdot}5$

(c) Verify the solution: $\dfrac{2^5}{2^{2\cdot5}} = \sqrt{32} \Rightarrow 2^{5-2\cdot5} = 2^{2\cdot5} \Rightarrow 2^{2\cdot5} = 2^{2\cdot5}$ ✓

(d) $\dfrac{\sqrt{128} - \sqrt{50}}{\sqrt{8}}$ Get factors of 128, 50 and 8 so that one factor is a square number.

$= \dfrac{\sqrt{64 \times 2} - \sqrt{25 \times 2}}{\sqrt{4 \times 2}}$ Rule: $\sqrt{a}\sqrt{b} = \sqrt{ab}$.

$= \dfrac{\sqrt{64}\sqrt{2} - \sqrt{25}\sqrt{2}}{\sqrt{4}\sqrt{2}}$

$= \dfrac{8\sqrt{2} - 5\sqrt{2}}{2\sqrt{2}}$ Simplify.

$= \dfrac{3\cancel{\sqrt{2}}}{2\cancel{\sqrt{2}}} = \dfrac{3}{2}$

Q7 $A = \pi(R^2 - r^2)$ Divide both sides by π.

$\Rightarrow \dfrac{A}{\pi} = (R^2 - r^2)$ Subtract R^2.

$\Rightarrow \dfrac{A}{\pi} - R^2 = -r^2$ Divide both sides by −1.

$\Rightarrow -\dfrac{A}{\pi} + R^2 = r^2$ Square root both sides.

$\Rightarrow r = \sqrt{R^2 - \dfrac{A}{\pi}}$

Q8 (a) $h = \dfrac{u^2}{2g}$ Multiply both sides by $2g$.

$\Rightarrow 2gh = u^2$ Take the square root of both sides.

$\Rightarrow u = \sqrt{2gh}$

(b) $h = 82{\cdot}45$ m and $g = 9{\cdot}8$ m/s^2.

$u = \sqrt{2gh} = \sqrt{2(9{\cdot}8)(82{\cdot}45)} = \sqrt{1616{\cdot}02} \approx 40{\cdot}2$ m/s

Chapter 12 – Applied Arithmetic 2

Q1 (a) (i) Convert \$510 to Euro:

Commission charged 3%: $0{\cdot}03 \times \$510 = \$15{\cdot}30$

Amount to be exchanged: $\$510 - \$15{\cdot}30 = \$494{\cdot}70$

\$494·70 to Euro:

Exchange rate: $€1 = \$1{\cdot}12$

$\$1{\cdot}12 = €1$

$\$\frac{1{\cdot}12}{1{\cdot}12} = €\frac{1}{1{\cdot}12}$

$\$1 = €\frac{1}{1{\cdot}12}$

Multiply both sides of the equation by the amount to be converted.

$\$1 \times 494{\cdot}70 = €\frac{1}{1{\cdot}12} \times 494{\cdot}70$

$\$494{\cdot}70 = €441{\cdot}6964\ldots$

$= €441{\cdot}70$

(ii) Convert €2100 to Chinese Yuan:

Commission charged 3% : $0{\cdot}03 \times €2100 = €63$

Amount to be exchanged: $€2100 - €63 = €2037$

€2037 to Chinese Yuan:

Exchange rate: $€1 = ¥7{\cdot}26$

Multiply both sides of the equation by the amount to be converted.

$€1 = ¥7{\cdot}26$

$€1 \times 2037 = ¥7{\cdot}26 \times 2037$

$€2037 = ¥14\,788{\cdot}62$

(iii) Convert 1750 Swedish Krona to Euro:

Commission charged 3% : $0{\cdot}03 \times \mathrm{kr}1750 = \mathrm{kr}52{\cdot}50$

Amount to be exchanged: $\mathrm{kr}1750 - \mathrm{kr}52{\cdot}50 = \mathrm{kr}1697{\cdot}50$

kr1697·50 to Euro:

Exchange rate: $€1 = \mathrm{kr}9{\cdot}46$

$\mathrm{kr}9{\cdot}46 = €1$

$\mathrm{kr}\frac{9{\cdot}46}{9{\cdot}46} = €\frac{1}{9{\cdot}46}$

$\mathrm{kr}1 = €\frac{1}{9{\cdot}46}$

Multiply both sides of the equation by the amount to be converted.

$$\text{kr}1 \times 1697{\cdot}50 = €\frac{1}{9{\cdot}46} \times 1697{\cdot}50$$

$$\text{kr}1697{\cdot}50 = €179{\cdot}4397\ldots$$

$$= €179{\cdot}44$$

Therefore, kr1697·50 is equal to €179·44.

(iv) Convert €2500 to Mexican Pesos:

Commission charged 3% : 0·03 × €2500 = €75

Amount to be exchanged: €2500 – €75 = €2425

€2425 to Mexican Pesos

Exchange rate: €1 = \$20·91

Multiply both sides of the equation by the amount to be converted.

€1 × 2425 = \$20·91 × 2425

€2425 = \$50 706·75

Therefore, €2425 is equal to \$50 706·75.

(b) **(i)** Sale price = 100% – 30% = 70%

70% = €85

$$1\% = €\frac{85}{70} = €1{\cdot}2143\ldots$$

100% = €1·2143 × 100

= €121·43

The original price of the shoes was €121·43.

(ii) Marked price = 100% = €70

$$1\% = €\frac{70}{100} = €0{\cdot}7$$

70% = 0·7 × 70 = €49

The sales price of the pair of jeans is €49.

Q2 **(a)** **(i)** Original cost:

118% = €530

$$1\% = €\frac{530}{118} = €4{\cdot}491\,525$$

$$\simeq €4{\cdot}4915$$

100% = €449·15

Original cost price was €449·15.

(ii) Profit made on the bike:

Selling price – cost price = €140 – €115 = €25

$$\% \text{ mark-up} = \frac{\text{profit}}{\text{cost price}} \times 100\%$$
$$= \frac{25}{115} \times 100\% = 21{\cdot}7391\% = 21{\cdot}7\% \text{ to 1 d.p.}$$

(iii) Cost of the treadmill:

Total cost – cost of the bike = €449·15 – €115 = €334·15

Selling price of the treadmill: €530 – €140 = €390

Profit on the treadmill:
Selling price – cost price = €390 – €334·15 = €55·85

$$\% \text{ margin} = \frac{\text{profit}}{\text{selling price}} \times 100\%$$
$$= \frac{55{\cdot}85}{390} \times 100\% = 14{\cdot}3205\% = 14{\cdot}32\%, \text{ correct to 2 d.p.}$$

(b) **(i)** Information given:

Cost price = €170

% mark-up = 28%

Substitute information into the formula:

$$\% \text{ mark-up} = \frac{\text{profit}}{\text{cost price}} \times 100\%$$
$$28\% = \frac{\text{profit}}{170} \times 100\%$$
$$\Rightarrow \frac{\text{profit}}{170} = \frac{28}{100}$$
$$\Rightarrow \text{profit} = \frac{28}{100} \times 170$$
$$= €47{\cdot}60$$

Selling price = cost price + profit
= €170 + €47·60
= €217·60

(ii) Selling price = €217·60

Profit = €47·60

$$\% \text{ margin} = \frac{\text{profit}}{\text{selling price}} \times 100\%$$
$$= \frac{47{\cdot}60}{217{\cdot}60} \times 100\%$$
$$= 21{\cdot}875\%$$
$$\simeq 22\%$$

Q3 **(a)** **(i)** **Shop A:**

1 bar : €4·08 ÷ 12 = €0·34

Shop B:

1 bar : €2·17 ÷ 7 = €0·31

(ii) Number of packets of bars required in Shop A: 84 ÷ 12 = 7 packets needed.

Cost of 7 packets of bars: €4·08 × 7 = €28·56

Number of packets of bars required in Shop B: 84 ÷ 7 = 12 packets needed.

Cost of 12 packets of bars: €2·17 × 12 = €26·04

Savings made by buying the bars in shop B: €28·56 – €26·04 = €2·52

(b) (i) Night rate

Units used = present reading – previous reading

= 41 323 – 40 451 = 872

Cost @ €0·083 45 per unit = 872 × €0·083 45 = €72·7684 ≃ €72·77

Day rate

Units used = present reading – previous reading

= 32 912 – 32 198 = 714

Cost @ €0·153 86 per unit = 714 × €0·153 86 = €109·856 04 ≃ €109·86

Total cost = €72·77 + €109·86 = €182·63

(ii) Total bill = Service charge + cost of units used + VAT @13·5%

= (€23 + €182·63) × 1·135

= €233·39005 = €233·39

Q4 (a)

Steps to calculating net income	Calculation
1 Standard rate of tax @ 20%	0·2 × €35 300 = €7060
2 Higher rate of tax @ 40% on balance	Balance = €57 000 – €35 300 = €21 700 Higher rate of tax: 0·4 × €21 700 = €8680
3 Gross tax due	€7060 + €8680 = €15 740
4 Tax credits = gross tax – tax paid	€15 740 – €14 090 = €1650

(b) (i)

Steps to calculating net income	Calculation
1 Standard rate of tax @ 20%	0·2 × €35 300 = €7060
2 Higher rate of tax @ 40% on balance	Balance = €39 700 – €35 300 = €4400 Higher tax = 0·4 × €4400 = €1760
3 Gross tax due	€7060 + €1760 = €8820
4 Tax payable = gross tax – tax credits	€8820 – €1650 = €7170
5 Net income / take-home pay = gross salary – deductions	€39 700 – €7170 = €32 530

(ii) New take-home pay: €35 470

Increase in take-home pay: €35 470 – €32 530 = €2940

As the higher rate of tax @ 40% on balance has been paid, we can state €2940 = 60% of increase in gross income.

Solve to find 100%:

$\frac{€2940}{60} = 1\%$ Divide by 60.

€49 = 1%

€49 × 100 = 100% Multiply by 100.

€4900 = 100%

New gross income = €39 700 + €4900 = €44 600.

Q5 (a) **(i)** Investment at the start of year 1 = €7600.

Final amount at end of year 1 = €7600 × 1·06 = €8056

(ii) Interest earned = final amount − investment

= €8660·20 − €8056

= €604·20

Interest rate (%) = $\frac{\text{interest}}{\text{principal}} \times 100\%$

$= \frac{604{\cdot}20}{8056} \times 100\% = 7{\cdot}5\%$

(b) **(i)** **Investment at the start of year 1** = €6800.

Interest earned: 0·045 × €6800 = €306

Tax @ 24% on interest: 0·24 × €306 = €73·44

Final amount at end of year 1 = €6800 + €306 − €73·44 = €7032·56

Investment at the start of year 2 = €7032·56.

Interest earned: 0·045 × €7032·56 = €316·465 = €316·47

Tax @ 24% on interest: 0·24 × €316·47 = €75·95

Final amount at end of year 2 = €7032·56 + €316·47 − €75·95
= €7273·08

(ii) Investment at the start of year 3 = €7273·08.

Investment at the end of year 3 = €7577·10.

Interest earned after tax @ 24%: = €7577·10 − €7273·08
= €304·02

To calculate the interest:

76% = €304·02

$1\% = \frac{€304{\cdot}02}{76} = €4{\cdot}0003$

100% = €4·0003 × 100 = €400·03

$$\text{Interest rate (\%)} = \frac{\text{interest}}{\text{principal}} \times 100\%$$

$$= \frac{400{\cdot}03}{7273{\cdot}08} \times 100\% = 5{\cdot}5\%$$

Chapter 13 – Fundamental Principles of Counting

Q1 As **'or'** is in the question we use the fundamental principle of counting 2 rule.

Total number of outcomes

= (number of laptops) **or** (number of smartphones)

= 4 + 3

= 7

Q2 (a) Number of possible different 'Lunch Specials'

= (sandwich choices) **and** (crisp choices) **and** (drink choices)

= (no. of sandwich choices) × (no. of crisp choices) × (no. of drink choices)

= 5 × 3 × 3

= 45

(b) Number of possible different 'Lunch Specials' without Cheese & Onion crisps

= (sandwich choices) **and** (crisp choices – Cheese & Onion) **and** (drink choices)

= (no. of sandwich choices) × (no. of crisp choices – 1) × (no. of drink choices)

= 5 × 2 × 3

= 30

Q3 The combination lock can use the following letters and numbers:

Letters = {A, B, C, D} Digits = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

Number of letters = 4 Number of digits = 10

So the number of possible codes for the combination lock

= (letter choices) **and** (digit choices)

= (number of letter choices) × (number of digit choices)

= 4 × 10

= 40

Q4 Number of possible outfits Conor can wear on holiday

= (T-shirts) **and** (shorts or jeans) **and** (sandals or trainers)

= (no. of T-shirts) × (no. of shorts or jeans) × (no. of sandals or trainers)

$= 5 \times (4 + 1) \times 2$

$= 5 \times 5 \times 2$

$= 50$

Q5 **(a)** The options for the spinner = {A, B, C, D} The options for the coin = {H, T}

Number of spinner options = 4 Number of coin options = 2

Number of possible outcomes for the game

= (spinner options) **and** (coin options)

$= 4 \times 2$

$= 8$

(b) **(i)** **Method 1**

List of outcomes:

(H, A), (H, B), (H, C), (H, D)

(T, A), (T, B), (T, C), (T, D)

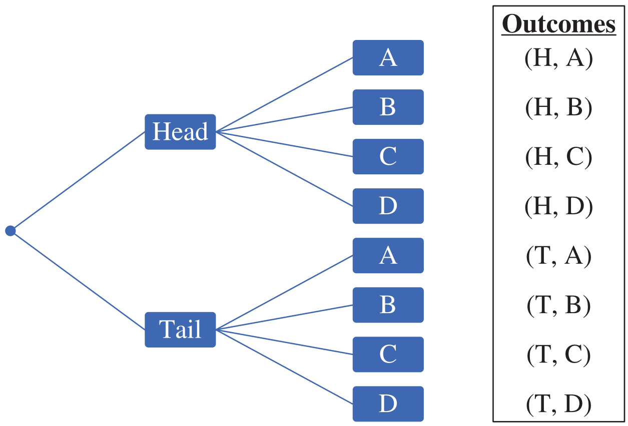
(ii) **Method 2**

Using a two-way table:

		Spinner			
		A	B	C	D
Coin	H	(H, A)	(H, B)	(H, C)	(H, D)
	T	(T, A)	(T, B)	(T, C)	(T, D)

(iii) **Method 3**

Using a tree diagram:



Q6 **(a)** Spinner options = {Red, Blue, Yellow, Green} Die options = {1, 2, 3, 4, 5, 6}

Number of spinner options = 4 Number of die options = 6

Number of possible outcomes for the game

= (spinner options) **and** (die options)

= 4 × 6

= 24

(b) **(i)** **Method 1**

List of outcomes:

(Red, 1), (Red, 2), (Red, 3), (Red, 4), (Red, 5), (Red, 6)
(Blue, 1), (Blue, 2), (Blue, 3), (Blue, 4), (Blue, 5), (Blue, 6)
(Yellow, 1), (Yellow, 2), (Yellow, 3), (Yellow, 4), (Yellow, 5), (Yellow, 6)
(Green, 1), (Green, 2), (Green, 3), (Green, 4), (Green, 5), (Green, 6)

(ii) **Method 2**

Using a two-way table:

		Die					
		1	2	3	4	5	6
Spinner	Red	(Red, 1)	(Red, 2)	(Red, 3)	(Red, 4)	(Red, 5)	(Red, 6)
	Blue	(Blue, 1)	(Blue, 2)	(Blue, 3)	(Blue, 4)	(Blue, 5)	(Blue, 6)
	Yellow	(Yellow, 1)	(Yellow, 2)	(Yellow, 3)	(Yellow, 4)	(Yellow, 5)	(Yellow, 6)
	Green	(Green, 1)	(Green, 2)	(Green, 3)	(Green, 4)	(Green, 5)	(Green, 6)

Q7 **(a)**

		Spinner			
		A	B	C	D
Die	1	(1, A)	(1, B)	(1, C)	(1, D)
	2	(2, A)	(2, B)	(2, C)	(2, D)
	3	(3, A)	(3, B)	(3, C)	(3, D)
	4	(4, A)	(4, B)	(4, C)	(4, D)
	5	(5, A)	(5, B)	(5, C)	(5, D)
	6	(6, A)	(6, B)	(6, C)	(6, D)

(b) Number of possible outcomes = (die options) **and** (spinner options)

= 6 × 4

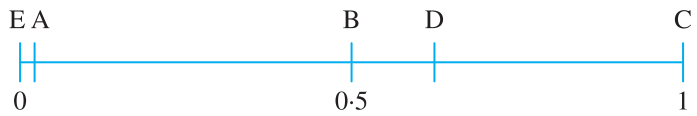
= 24

(c) Outcomes consisting of an odd number and B are (1, B) and (3, B) and (5, B). So, the number of outcomes containing an odd number and B = 3.

Chapter 14 – Probability

Q1 (a)

Event	Probability
A card is picked at random from a standard deck of playing cards. A = Ace of hearts is picked	$\frac{1}{52}$
A fair coin is tossed. B = a tail is the outcome for the toss	$\frac{1}{2}$
A day is chosen at random from the list of the days of the week. C = the day contains the letter **a**	$\frac{7}{7} = 1$
A month is chosen at random from the list of months of the year. D = the month contains the letter **r**	$\frac{8}{12} = \frac{2}{3}$
A letter is picked at random from the word ENORMOUS. E = the letter is **W**	$\frac{0}{8} = 0$



(b) Rolling an odd number on a die.

(c) Selecting a positive number from the natural numbers.

(d) Selecting a black heart from a standard deck of cards.

Q2 (a) The total number of shirts in the shop is 18.

(b)

Shirt size	S	M	L	XL
Frequency	3	7	6	2

(c) $P(\text{large shirt}) = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}} = \frac{6}{18} = \frac{1}{3}$

(d) There are now 17 shirts to choose from.

$P(\text{small shirt}) = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}} = \frac{3}{17}$

Q3 (a) $x = 106 - (34 + 16 + 6 + 11 + 32) = 106 - 99 = 7$

(b) $P(\text{girl who ate at the Chinese outlet}) = \frac{32}{106} = \frac{16}{53}$

(c) $P(\text{boy}) = \frac{56}{106} = \frac{28}{53}$

(d) $P(\text{girl}) = 1 - P(\text{boy}) = 1 - \frac{28}{53} = \frac{25}{53}$

(e) $P(\text{student ate in the burger outlet}) = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}} = \frac{45}{106}$

Q4 (a) Relative frequency of manufacturing a faulty part $= \frac{4}{360} = \frac{1}{90}$

(b) (i) Expected frequency = relative frequency × the number of trials

$$= \frac{1}{90} \times 500 = 5{\cdot}\dot{5}$$

As we can only have whole numbers of products, the number of faulty parts expected is 6.

(ii) Expected frequency = relative frequency × the number of trials

$$= \frac{1}{90} \times 1350 = 15$$

The number of faulty parts expected is 15.

(iii) Expected frequency = relative frequency × the number of trials

$$= \frac{1}{90} \times 20\,000 = 222{\cdot}\dot{2}$$

As we can only have whole numbers of products, the number of faulty parts expected is 223.

Q5 (a) Relative frequency of selecting a red card $= \frac{17}{30} = 0{\cdot}57$

(b) Theoretical probability of a red card being selected

$$= \frac{\text{number of favourable outcomes that exist}}{\text{total number of possible outcomes}} = \frac{26}{52} = 0{\cdot}5.$$

(c) The answers are different as relative frequency is based on experimental data. In this situation the number of trials was small.

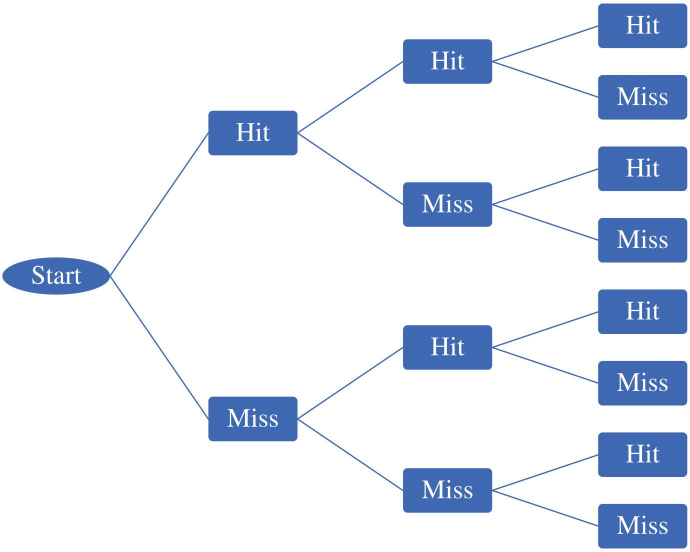
(d) Recall that the relative frequency approaches the theoretical probability when there are a very large number of trials. So to produce a relative frequency that is closer to the theoretical probability the number of trials carried out would need to be increased.

Q6 (a) $P(\text{arrow hits bullseye}) = 0{\cdot}4.$

So $P(\text{arrow does not hit bullseye})$

$= 1 - P(\text{arrow hits bullseye}) = 1 - 0{\cdot}4 = 0{\cdot}6$

So the tree diagram can be drawn to represent this information:



Outcomes

(H, H, H) $= 0{\cdot}4 \times 0{\cdot}4 \times 0{\cdot}4 = 0{\cdot}064$

(H, H, M) $= 0{\cdot}4 \times 0{\cdot}4 \times 0{\cdot}6 = 0{\cdot}096$

(H, M, H) $= 0{\cdot}4 \times 0{\cdot}6 \times 0{\cdot}4 = 0{\cdot}096$

(H, M, M) $= 0{\cdot}4 \times 0{\cdot}6 \times 0{\cdot}6 = 0{\cdot}144$

(M, H, H) $= 0{\cdot}6 \times 0{\cdot}4 \times 0{\cdot}4 = 0{\cdot}096$

(M, H, M) $= 0{\cdot}6 \times 0{\cdot}4 \times 0{\cdot}6 = 0{\cdot}144$

(M, M, H) $= 0{\cdot}6 \times 0{\cdot}6 \times 0{\cdot}4 = 0{\cdot}144$

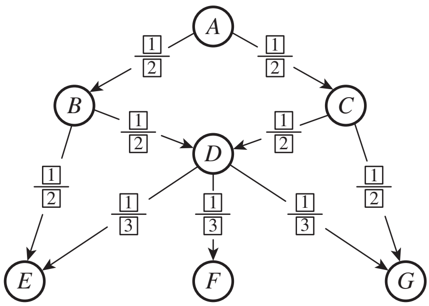
(M, M, M) $= 0{\cdot}6 \times 0{\cdot}6 \times 0{\cdot}6 = 0{\cdot}216$

(b) P(all three arrows hit the bullseye) $= 0{\cdot}4 \times 0{\cdot}4 \times 0{\cdot}4 = 0{\cdot}064$

(c) P(none of the three arrows hit the bullseye) $= 0{\cdot}6 \times 0{\cdot}6 \times 0{\cdot}6 = 0{\cdot}216$

(d) P(at least one arrow hits the bullseye)
$= 1 - P$(none of the arrows hit the bullseye) $= 1 - 0{\cdot}216 = 0{\cdot}784$

Q7 (a)



(b) (i)
1. A → B → E
2. A → B → D → E
3. A → C → D → E

3 ways

(ii)
1. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
2. $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{3} = \frac{1}{12}$
3. $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{3} = \frac{1}{12}$

Probability $= \frac{1}{4} + \frac{1}{12} + \frac{1}{12} = \frac{5}{12}$

Chapter 15 – Statistics 1: Statistics Use, Data Types and the Data Handling Cycle

Q1 (a) Statistics is about the collection, organisation, presentation and interpretation of data.

(b) Data is individual facts, statistics or items of information.

(c) Univariate data looks at one item of data at a time from each topic, for example, height.

(d) The population is the entire group of people, animal or things about which we want information.

(e) A sample is a subset of a population, which we actually collect information from in order to draw conclusions about the whole population.

(f) A sample is **biased** if individuals or groups from the population are not represented in the sample.

(g) In a random sample each member of the population has an equal chance of being selected.

(h) Categorical/Qualitative data is data that can be described using words only. It can be ordered or unordered.

(i) Numerical/Quantitative data is data that can be represented by numbers. It can be discrete or continuous.

Q2

Type of data	Definition	Examples	Suitable graphical representation
Categorical	Data that can be described using words only. Can be ordered or unordered.	• Places in a race: 1st, 2nd, 3rd, etc. • Months of the year • Days of the week • Grades in an exam • Clothes sizes: extra-small, small, medium, large, etc. • Favourite colour • Film genre • Animals • Favourite food	• Bar chart • Line plot • Pie chart
Numerical	Data that can be represented by numbers. Can be discrete or continuous.	• Family size • Shoe size • Number of pens in your pencil case • Shirt collar size • Number of goals scored • Height • Age • Time • Temperature • Weight • Area • Length	• Bar chart • Line plot • Pie chart • Stem and leaf plot • Histogram

Q3

Data type	How it is collected	Advantages	Disadvantages
Primary data is data collected by the person who uses it.	Is collected by means of a survey. The different types of surveys are: **1** Questionnaires **2** Experimental study: • the researcher deliberately influences events and investigates the effects of the intervention Examples: • Laboratory experiment • Clinical trial **3** Observational study: • the researcher collects information but does not influence events Example: • Monitoring behaviour	• You collect the data you need • It's accurate • Easy to understand	• Takes time • Need the help of other people • Can be expensive
Secondary Data is data collected by another person	• Books • Magazines • Newspapers • Internet • TV • Central Statistics Office (CSO) • Census at school	• Cheap to collect • Doesn't take a lot of time	• It may not be up to date • May not be accurate • May not provide the information needed • May be biased

Q4 In an experimental study the researcher deliberately influences events and investigates the effects of the intervention. In an observational study the researcher collects information but does not influence events.

Q5

Method	Advantages	Disadvantages
Face to face	People are more likely to answer. More difficult questions can be asked and explained if needed.	It takes a long time and is not random. The interviewee is more likely to lie. Expensive.
Phone	Almost everyone has a phone. Questions can be explained. It's random.	Phone calls can be expensive. It is difficult to get people's phone numbers. Calling people randomly can annoy them.
Post	People might have more time to answer questions at home. Not expensive.	People may not return their questionnaires. Questions cannot be explained. May not be representative of the population.
Website	Saves time. Very low cost. Easy to carry out.	Questions cannot be explained. Only people who want to answer will take part.

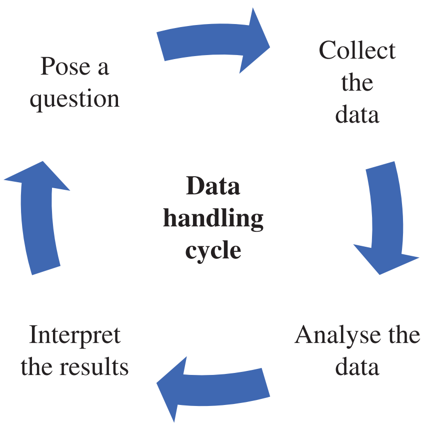
Q6 The data handling cycle follows these steps:

1 Pose a question

2 Collect the data

3 Analyse the data

4 Interpret the results



Q7 Population: All Dublin inner city residents.

Variable measured: opinion on the police service, for example by a rating scale.

Sample: 300 adults living at the 300 addresses chosen (not given any information on the response rate).

Potential bias: may overestimate positive feedback on police service because a Garda is asking the questions – would be better to have someone neutral or trusted by the community to carry out the survey. Would also need information on the response rate.

Q8 (a)

Question	Numerical	Categorical
How many Geography classes do you teach each week?	✓	
How much do you like teaching Geography? A lot ☐ A little ☐ Not at all ☐		✓
What subjects (other than Geography) do you teach?		✓

(b) Eithne could select a simple random sample from all the post-primary schools in Ireland by:

- Step 1: Getting a list of all of the post-primary schools in Ireland.
- Step 2: Randomly selecting a number of them, e.g. by using a random number generator.

(c) Advantages of using email to collect data include that it is quick, convenient and cheap.
Disadvantages include that not everyone has email, the mail may go to spam, there is a faulty computer or people don't reply.

Q9

Question	Type of data
(a) What is your height in centimetres?	Numerical
(b) What subjects do you study at school?	Categorical

Chapter 16 – Statistics 2: Central Tendency and Spread of Data

Q1 (a) (i) The mean is found by adding (summing) all the values together and dividing by the number of values in the data set.

$$\text{Mean} = \frac{\text{sum of all the values}}{\text{number of values}}$$

$$= \frac{102 + 108 + 107 + 109 + 108 + 102 + 110}{7} = \frac{746}{7} = 106{\cdot}571...$$

The mean is 106·57, accurate to 2 decimal places.

(ii) The mode is the data value that occurs most often in the set of data.

102, **108**, 107, 109, **108**, **102**, 110

In this distribution, 102 and 108 both appear twice, so there are two modes for this set of data.

The modes are 102 and 108.

(iii) The median is the middle number when the data is ranked:

Rank the data: 102, 102, 107, 108, 108, 109, 110

Identify/find the middle number: 102, 102, 107, **108**, 108, 109, 110

The median is the 4th data value. The median is 108.

(iv) The range = largest value – smallest value = 110 − 102 = 8

(b) As there are no outliers in this data set, the mean or median values can be used. As there are two values for the mode, these values should not be used.

Q2 (a) (i) $$\text{Mean} = \frac{\text{sum of all the values}}{\text{number of values}}$$

$$= \frac{11 + 4 + 6 + 8 + 3 + 10 + 8 + 10 + 4 + 12 + 13}{11} = \frac{89}{11} = 8{\cdot}09...$$

(ii) The modes are the data values that occur most often in the set of data. There are three modes for this data set: 4, 8 and 10, which all occur twice.

(iii) The median is the middle value, when the data is ranked. This is the 6th value, which is 8.

3, 4, 4, 6, 8, 8, 10, 10, 11, 12, 13

Median = 8

(iv) The range = largest value – smallest value = 13 − 3 = 10

(b) Ignoring outliers, the median is a better value to use to describe the central tendency in this case. This value exists in the data set and is close to the mean.

Q3 (a) On average, the data in set C are the biggest numbers and the data in set D are the smallest numbers. We know this as set C has the biggest mean and set D has the smallest mean, so these numbers contain the biggest and smallest numbers.

(b) The set that contains more numbers than any other is D and the set that contains fewer numbers than any other is A. The size of the data set is given by n.

(c) Total of set A = 12 × 15 = 180

Total of set B = 50 × 15 = 750

Total of set C = 50 × 55 = 2750

Total of set D = 500 × 5 = 2500

So the set with the greatest total is C.

(d) The data in set B have the greatest difference between their highest and lowest values. This is because the range is the biggest value.

Q4 (a) $\text{Mean} = \dfrac{\text{sum of all the values}}{\text{number of values}}$

$$\therefore \text{Mean} = \frac{1+x+4+3}{4} = 2 \Rightarrow \frac{x+8}{4} = 2 \Rightarrow x+8 = 8 \Rightarrow x = 0$$

The number of goals scored in the second game was 0 goals.

(b) **(i)** To find the mean, use the formula: mean = $\mu = \dfrac{\sum fx}{\sum f}$.

Number of days absent x	**Frequency** f	**Number of days absent × Frequency** fx
0	7	(0)(7) = 0
1	9	(1)(9) = 9
2	11	(2)(11) = 22
3	12	(3)(12) = 36
4	7	(4)(7) = 28
5	4	(5)(4) = 20
	$\sum f = 50$	$\sum fx = 115$

Multiply the frequencies by the corresponding number of days absent, fx

Sum of the product of the frequencies and the corresponding number of days absent, $\sum fx$

Sum of the frequencies, $\sum f$

$$\therefore \text{Mean} = \mu = \frac{\sum fx}{\sum f} = \frac{115}{50} = 2{\cdot}3.$$

(ii) The mode is the most common number of days absent. This is 3 days.

(iii) The median number of days that the employees were absent is the middle data value, which is between the 25th and 26th data value. The median number of days absent is 2 days.

(iv) The range = largest value – smallest value = 5 − 0 = 5.

(v) As there are no outliers in this data set, use the mean which is the most common measure of centre. The mean is 2·3 days.

Q5 (a)

Salary (€1000s)	0–10	10–20	20–30	30–40	40–50	50–60	60–70
Number of employees	1	6	12	9	2	1	1

(b)

Salary (€1000s)	**Number of employees Frequency f**	**Mid-interval value x**	**Frequency × mid-interval value fx**
0–10	1	5	(1)(5) = 5
10–20	6	15	(6)(15) = 90
20–30	12	25	(12)(25) = 300
30–40	9	35	(9)(35) = 315
40–50	2	45	(2)(45) = 90
50–60	1	55	(1)(55) = 55
60–70	1	65	(1)(65) = 65
	$\sum f = 32$		$\sum fx = 920$

$$\therefore \text{ Mean} = \mu = \frac{\sum fx}{\sum f} = \frac{920}{32} = 28{\cdot}75$$

Therefore, the mean salary earned is €28 750.

(c) **(i)** Another method which could have been used to calculate the mean salary is to add up all the actual salaries given in the question and divide the answer by 32.

(ii) The second method is more accurate as the mean achieved gives the actual mean for the given salaries. The mid-interval values give a good approximation only.

$$\text{Mean} = \frac{\text{salaries total}}{\text{number of employees}} = \frac{€917\,000}{32} = €28\,656{\cdot}25$$

Q6 (a) Mean $= \dfrac{\text{sum of all the values}}{\text{number of values}}$

$\therefore$ Mean $= \dfrac{13+6+5+x+7}{5} = 8 \Rightarrow \dfrac{x+31}{5} = 8 \Rightarrow x = 40 - 31 = 9$

(b) **(i)** Create a frequency table to find the mean weight of the Junior Cycle students.

Weight in kg	Number of students Frequency f	Mid-interval value x	Frequency × mid-interval value fx
40–45	7	42·5	(7)(42·5) = 297.5
45–50	9	47·5	(9)(47·5) = 427.5
50–55	22	52·5	(22)(52·5) = 1155
55–60	27	57·5	(27)(57·5) = 1552.5
60–65	24	62·5	(24)(62·5) = 1500
65–70	28	67·5	(28)(67·5) = 1890
70–75	8	72·5	(8)(72·5) = 580
	$\sum f = 125$		$\sum fx = 7402{\cdot}5$

$\therefore$ Mean $= \mu = \dfrac{\sum fx}{\sum f} = \dfrac{7402{\cdot}5}{125} = 59{\cdot}22$

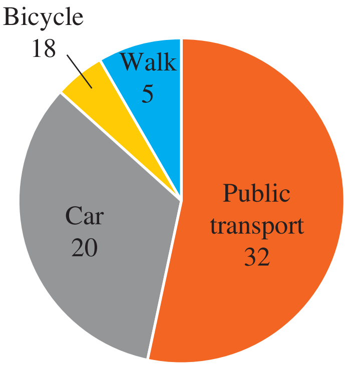
(ii) The modal weight of the students is in the range 65–70 kg.

(iii) The median weight of the students is the 63rd data value, which is in the range 55–60 kg.

(iv) The median is the measure of centre which is the most appropriate to use for this data, because extreme outliers exist.

Chapter 17 – Statistics 3: Representation and Interpretation of Statistics

Q1 **(a)**



We know that 60 students = 360° ⇒ 1 student = $\frac{360°}{60} = 6°$

The 'public transport' angle = $32 \times 6° = 192°$

The 'car' angle = $20 \times 6° = 120°$

The 'bicycle' angle = $3 \times 6° = 18°$

The 'walk' angle = $5 \times 6° = 30°$

(b) This is an appropriate method to display the data as the angles are not close in size and there are not too many categories.

(c) The percentage of students who travel to school by public transport $= \frac{32}{60} \times 100 = 53{\cdot}\dot{3} = 53\%$ to the nearest whole number.

Q2 Use the pie chart to calculate the number of hours spent on different activities in 24 hours.

The number of hours spent sleeping $= \frac{120}{360} \times 24 = 8$ hours

The number of hours spent at school $= \frac{90}{360} \times 24 = 6$ hours

The number of hours spent doing homework $= \frac{45}{360} \times 24 = 3$ hours

The number of hours spent eating meals $= \frac{360 - 120 - 90 - 45 - 75}{360} \times 24$

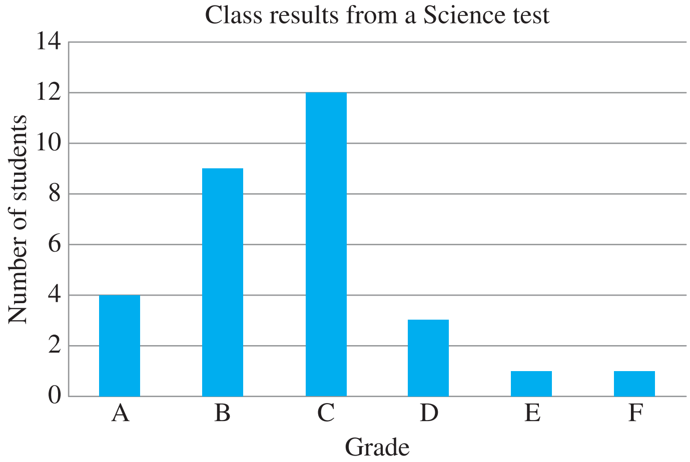
$= \frac{30}{360} \times 24 = 2$ hours

The number of hours spent enjoying leisure activities $= \frac{75}{360} \times 24 = 5$ hours

Activity	Sleeping	School	Homework	Meals	Leisure
Number of hours	8	6	3	2	5

Q3 (a) 30 students did the class test.

(b)



(c) The modal grade is C, as it is the grade that most students achieved in the test.

(d) The percentage of students who got a grade C or above

$= \dfrac{\text{number of students who got a grade C or above}}{\text{number of students who sat the test}} \times 100 = \dfrac{25}{30} \times 100$

$= 83{\cdot}\dot{3}\%$

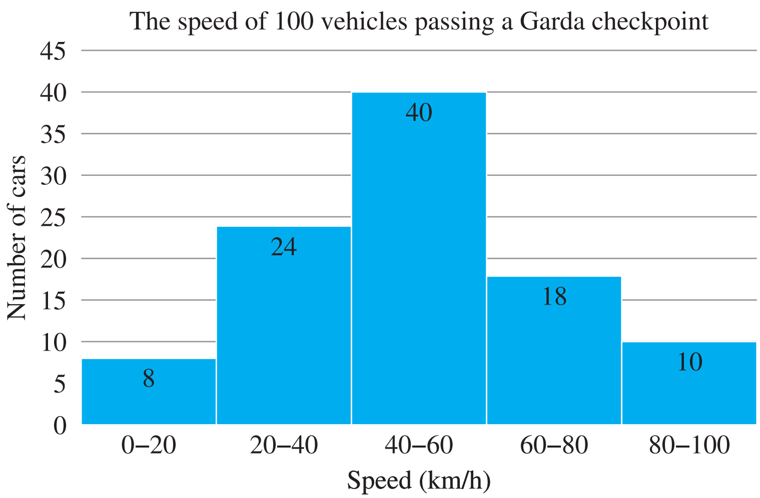
Q4 (a) Taking mid-interval values of the frequency table becomes:

Speed in km/h	0–20	20–40	40–60	60–80	80–100	Total
Mid-interval speed (x)	10	30	50	70	90	
Number of cars (f)	8	24	40	18	10	$\sum f = 100$
$f(x)$	80	720	2000	1260	900	$\sum f(x) = 4960$

$\text{Mean} = \dfrac{\text{sum of all the values}}{\text{number of values}} = \dfrac{\sum f(x)}{\sum f} = \dfrac{4960}{100} = 49{\cdot}6$

Hence, the mean speed of the cars passing the checkpoint is 49·6 km/h

(b)

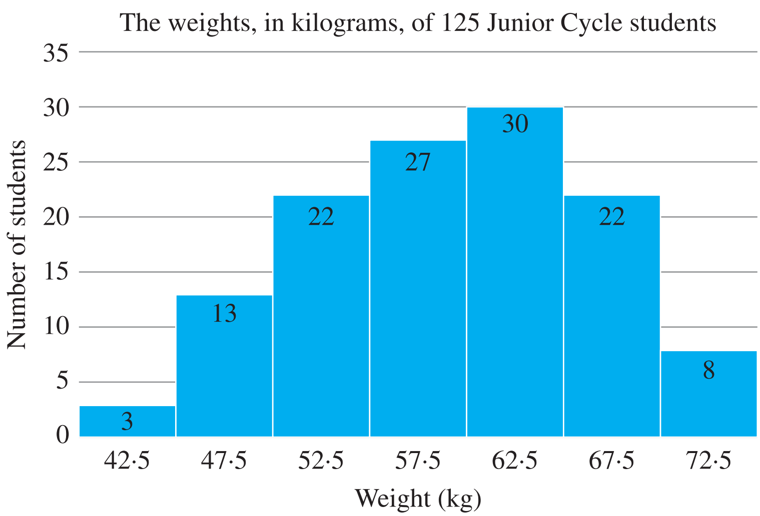


Q5 (a) Taking mid-interval values of the frequency table becomes:

Weight in kg	40–45	45–50	50–55	55–60	60–65	65–70	70–75	Total
Mid-interval weight (x)	42·5	47·5	52·5	57·5	62·5	67·5	72·5	
Number of students (f)	3	13	22	27	30	22	8	$\sum f = 125$
$f(x)$	127·5	617·5	1155	1552·5	1875	1485	580	$\sum f(x) =$ 7392·5

$$\text{Mean} = \frac{\text{sum of all the values}}{\text{number of values}} = \frac{\sum f(x)}{\sum f} = \frac{7392 \cdot 5}{125} = 59 \cdot 14 \text{ kg}$$

(b)



Q6 **(a)** 20 people attended the meeting.

(b) The range = maximum age – minimum age = 69 − 18 = 51 years.

(c) The median age is half way between 44 and 45. Therefore the median is 44·5 years old.

Q7 **(a)**

Aerobics class		Swimming class
9	0	7 8
8 7 5 3	1	2 3 4 5 6 8
7 4 4 2 0	2	2 2 3 4 4 4 9
9 7 7 7 3 2	3	1 3 5 6 6 8
9 8 8 5 2 2 1	4	1 2 5 7 8
8 6 4 3 1	5	1 2 3
3 1	6	2
Key : 1|5 means 15		

(b) Aerobics median: $\dfrac{37 + 39}{2} = 38$ years old.

Swimming median: $\dfrac{29 + 31}{2} = 30$ years old.

(c) Another measure of central tendency which could have been used when examining this data is either the mean or the mode.

(d) One observation that can be made from the data is that an older age group take the aerobics class (or a younger age group take the swimming class).

Chapter 18 – Geometry 1: Theorems, Axioms and Corollaries

Q1 Corollary 3: Each angle in a semi-circle is a right-angle; or Corollary 4: If the angle standing on a chord $[BC]$ at some point of the circle is a right-angle, then $[BC]$ is a diameter.

Q2

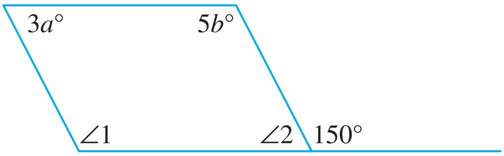
1 Axiom	An axiom is a statement accepted without proof.
2 Theorem	A theorem is a statement which can be proved from the axioms by logical argument.
3 Corollary	A corollary is a statement which can be made following a given theorem.

Q3

1 Proof	A proof involves writing well-structured, logical steps that use axioms and previously proved theorems to arrive at a conclusion about a statement.
2 Collinear	If three or more points lie on the same line, they are collinear.
3 Congruent	Two triangles are congruent if all the sides and angles of one triangle are equal to the corresponding sides and angles of the other triangle.

Chapter 19 – Geometry 2: Applications of Theorems, Axioms and Corollaries

Q1 Labelling the unknown angles:



$|\angle 1| = 150°$ … (Corresponding angles as opposite sides of a parallelogram are parallel)

$5b° = 150°$ … (Opposite angles of a parallelogram are equal)

$\Rightarrow b° = \dfrac{150°}{5} = 30°$

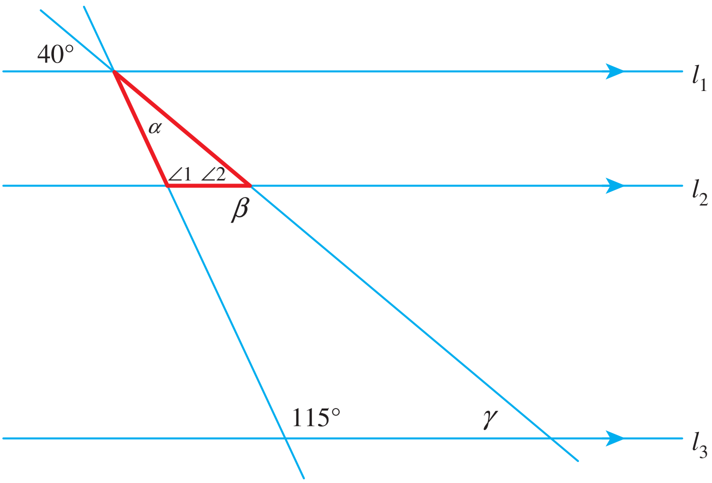
$|\angle 2| + 150° = 180°$ … (Straight line)

$|\angle 2| = 180° - 150° = 30°$

$3a° = 30°$ … (Opposite angles of a parallelogram are equal)

$\Rightarrow a° = \dfrac{30°}{3} = 10°$

Q2



Considering the triangle highlighted:

$\alpha + |\angle 1| + |\angle 2| = 180°$

$|\angle 1| = 115°$ ... (Corresponding angles)

$|\angle 2| = 40°$ ... (Corresponding angles)

$\alpha + 115° + 40° = 180°$

$\therefore\ \alpha = 180° - (115° + 40°)$

$\therefore\ \alpha = 180° - 155° = 25°$

$\beta = 180° - 40° = 140°$ ... (Straight line)

$\gamma = 40°$ ... (Corresponding angles with $|\angle 2| = 40°$)

Q3 **(a)** Given that the angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on the same arc:

$|\angle SOP| = 2 \times 32° = 64°$

(b) $|\angle SRP| = |\angle SQP| = 32°$ ... (Angles standing on the same arc)

Q4 Since $[DE]$ is parallel to $[CB]$:

$$\frac{|AB|}{|AE|} = \frac{|AC|}{|AD|}$$

Filling in the given information:

$$\frac{|AB|}{5} = \frac{14}{4}$$

$$5\left(\frac{|AB|}{5}\right) = 5\left(\frac{14}{4}\right)$$

$$|AB| = 5\left(\frac{14}{4}\right) = \frac{70}{4}$$

$\therefore\ |AB| = 17{\cdot}5$

From the diagram, $|EB| = |AB| - |AE|$.

$|EB| = 17{\cdot}5 - 5 = 12{\cdot}5$

Q5 Consider triangle ABC

$\angle ABC = 90°$ ... (Angle in a semi-circle)

Consider triangle BCD

$|\angle BCD| = 90°$ ... (Angle in a semi-circle)

Consider triangle ACD

$|\angle ADC| = 90°$ ... (Angle in a semi-circle)

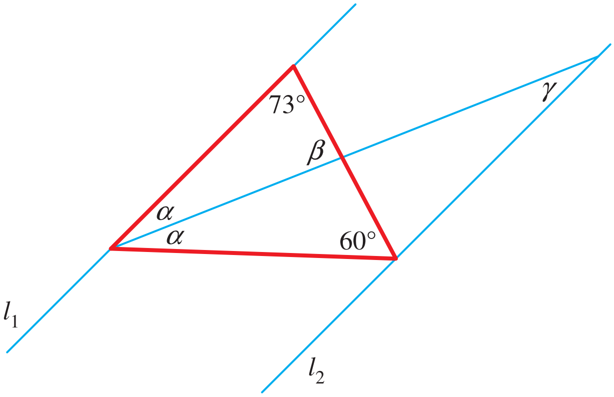
Consider triangle ABD

$|\angle BAD| = 90°$ ... (Angle in a semi-circle)

$\therefore$ $ABCD$ is a rectangle.

Q.E.D.

Q6



Considering the triangle highlighted we can state that:

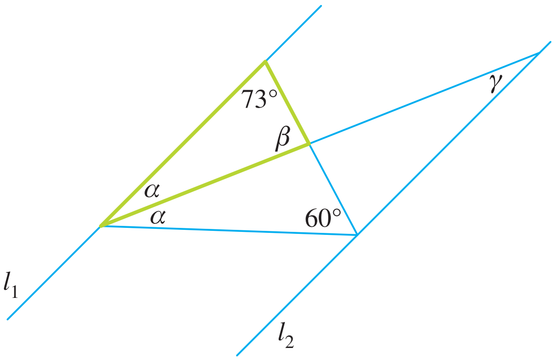
$73° + 60° + 2\alpha = 180°$ ... (Sum of the angles of a triangle)

$133° + 2\alpha = 180°$

$2\alpha = 180° - 133°$

$2\alpha = 47°$

$\alpha = \dfrac{47°}{2} = 23.5°$



Considering the triangle highlighted:

$73° + \alpha + \beta = 180°$ ... (Sum of the angles of a triangle)

$73° + 23{\cdot}5° + \beta = 180°$

$96{\cdot}5° + \beta = 180°$

$\beta = 180° - 96{\cdot}5°$

$\beta = 83{\cdot}5°$

Given that $l_1 \parallel l_2$:

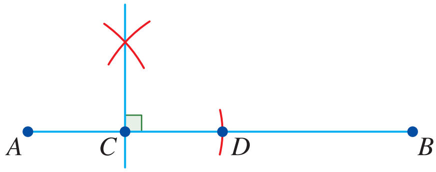
$\alpha = \gamma$

$\Rightarrow \gamma = 23{\cdot}5°$ (Alternate angles)

Chapter 20 – Geometry 3: Constructions

Q1 (a)

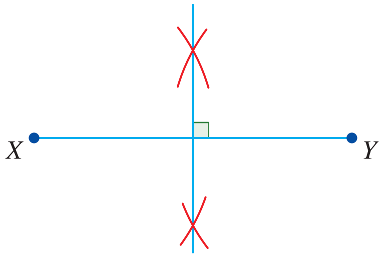
(b)



See Construction 4 for step-by-step instructions on how to construct this diagram.

Q2 (a)

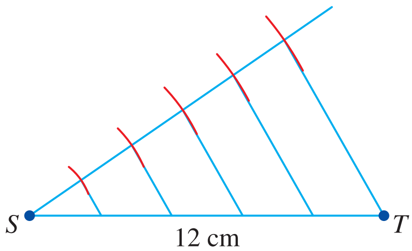
(b)



See Construction 2 for step-by-step instructions on how to construct this diagram.

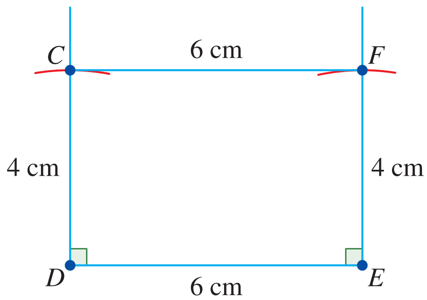
Q3 (a)

(b)



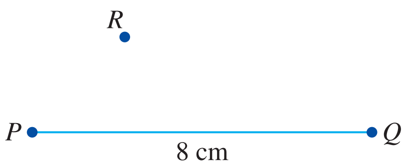
See Construction 7 for step-by-step instructions on how to construct this diagram.

Q4

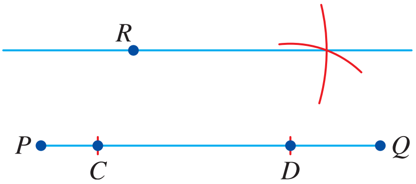


See Construction 15 for step-by-step instructions on how to construct this diagram.

Q5 (a)



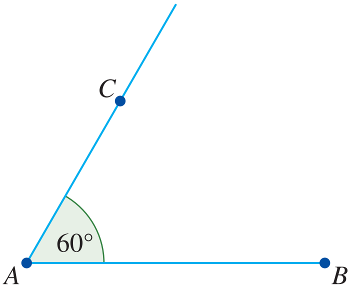
(b)



See Construction 5 for step-by-step instructions on how to construct this diagram.

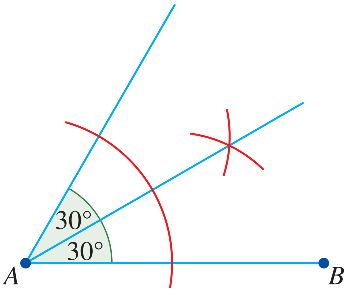
Q6 (a)

(b)



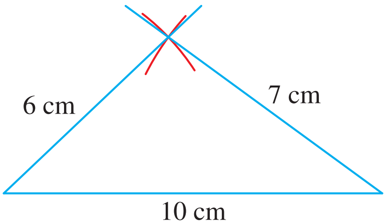
See Construction 9 for step-by-step instructions on how to construct this diagram.

(c)



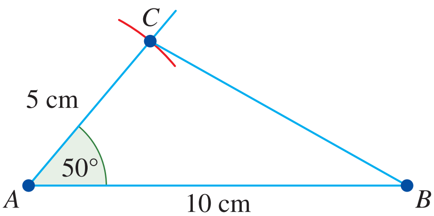
See Construction 1 for step-by-step instructions on how to construct this diagram.

Q7 (a)



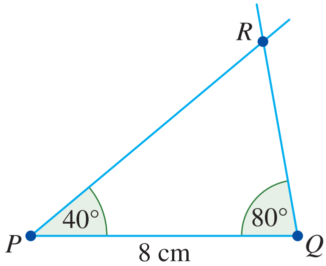
See Construction 10 for step-by-step instructions on how to construct this diagram.

(b)



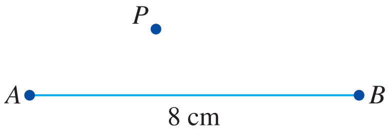
See Construction 11 for step-by-step instructions on how to construct this diagram.

(c)

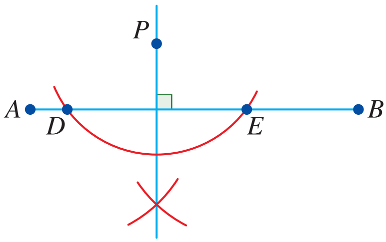


See Construction 12 for step-by-step instructions on how to construct this diagram.

Q8 (a)

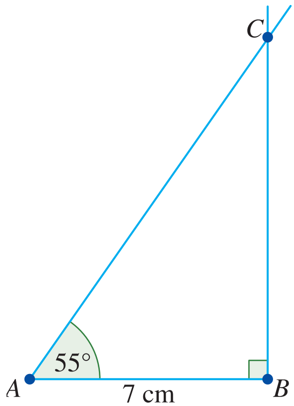


(b)



See Construction 3 for step-by-step instructions on how to construct this diagram.

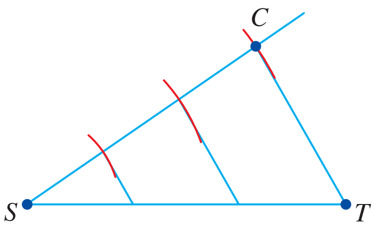
Q9



See Construction 14 for step-by-step instructions on how to construct this diagram.

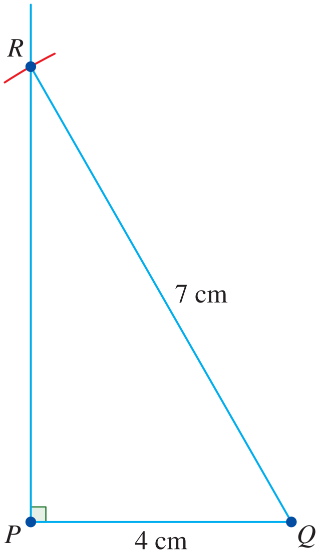
Q10 (a)

(b)



See Construction 6 for step-by-step instructions on how to construct this diagram.

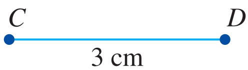
Q11

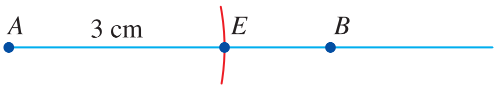


See Construction 13 for step-by-step instructions on how to construct this diagram.

Q12 (a)

(b)

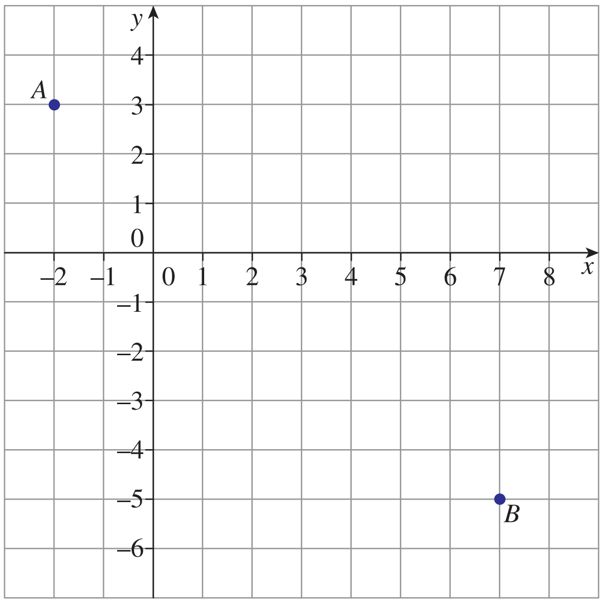




See Construction 8 for step-by-step instructions on how to construct this diagram.

Chapter 21 – Coordinate Geometry of the Line

Q1 (a)



(b) Label one of the points, say A, as (x_1, y_1) and the other as $B(x_2, y_2)$.

$$\begin{array}{cc} (x_1, y_1) & (x_2, y_2) \\ \Updownarrow \; \Updownarrow & \Updownarrow \; \Updownarrow \\ A(-2,\ 3) & B(7, -5) \end{array}$$

Substituting this information into the distance formula and simplifying:

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(7 - (-2))^2 + (-5 - 3)^2}$$
$$= \sqrt{(9)^2 + (-8)^2} = \sqrt{81 + 64} = \sqrt{145}$$

(c) Label one of the points, say A, as (x_1, y_1) and the other as $B(x_2, y_2)$. Call the midpoint point P.

$$\begin{array}{cc} (x_1, y_1) & (x_2, y_2) \\ \Updownarrow \; \Updownarrow & \Updownarrow \; \Updownarrow \\ A(-2,\ 3) & B(7, -5) \end{array}$$

Substituting this information into the midpoint formula:

$$P = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{-2 + 7}{2}, \frac{3 + (-5)}{2}\right)$$
$$= \left(\frac{5}{2}, \frac{-2}{2}\right)$$
$$= (2{\cdot}5, -1)$$

Q2 (a) Label one of the points, say X, as (x_1, y_1) and the other as $Y(x_2, y_2)$.

$$\begin{array}{cc} (x_1,\ y_1) & (x_2, y_2) \\ \Updownarrow, \; \Updownarrow & \Updownarrow, \Updownarrow \\ X(2, -4) & Y(6, 8) \end{array}$$

Substituting this information into the slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{8 - (-4)}{6 - 2}$$
$$= \frac{12}{4} = 3$$

(b) The slope of the line, m, is 3.

A point on the line is, say, $X = (2, -4)$.

Substitute the value of the slope and the point coordinates into the formula $y - y_1 = m(x - x_1)$.

$$y - (-4) = 3(x - 2)$$
$$y + 4 = 3x - 6$$

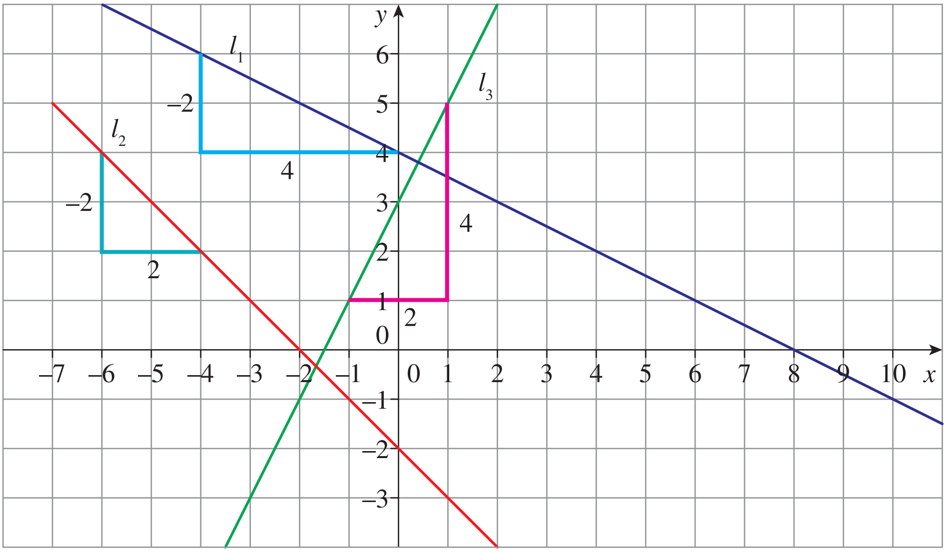
$$-3x + y + 4 + 6 = 0$$

$$-3x + y + 10 = 0$$

or

$$3x - y - 10 = 0$$

Q3 (a) The graph can be used to find the 'rise' and 'run' figures and the y-intercepts. Then in each case the values for m and c can be substituted into the equation $y = mx + c$ to obtain the equation of the line.



Line l_1

Slope $m = \frac{\text{rise}}{\text{run}} = \frac{-2}{4} = -\frac{1}{2}$ and y-intercept (0, 4).

Substituting into the equation of a line in the form $y = mx + c$

$$\Rightarrow l_1 : y = -\frac{1}{2}x + 4$$

$$\Rightarrow l_1 : 2y = -x + 8$$

Line l_2

Slope $m = \frac{\text{rise}}{\text{run}} = \frac{-2}{2} = -1$ and y-intercept (0, −2).

Substituting into the equation of a line in the form $y = mx + c$

$$\Rightarrow l_2 : y = -x - 2$$

Line l_3

Slope $m = \frac{\text{rise}}{\text{run}} = \frac{4}{2} = 2$ and y-intercept (0,3).

Substituting into the equation of a line in the form $y = mx + c$

$$\Rightarrow l_3 : y = 2x + 3$$

(b) The line with the greatest slope is $l_3 : y = 2x + 3$ as 2 represents the greatest slope.

(c) $l_1 : y = -\frac{1}{2}x + 4$ and $l_3 : y = 2x + 3$ are perpendicular, because when their slopes are multiplied together, the result is –1.

$$m_{l_1} \times m_{l_2} = \left(-\frac{1}{2}\right) \times (2) = \frac{-2}{2} = -1$$

Q4 **(a)** To find the slope, we can rearrange the equation $ax + by + c = 0$ to make y the subject of the equation in the form $y = mx + c$.

Line 1	Line 2	Line 3
$-x + 2y - 3 = 0$ $2y = x + 3$ $\frac{2y}{2} = \frac{1}{2}x + \frac{3}{2}$ $y = \frac{1}{2}x + \frac{3}{2}$ $\therefore$ Slope $(m_C) = \frac{1}{2}$	$4x + 2y - 3 = 0$ $2y = -4x + 3$ $\frac{2y}{2} = \frac{-4}{2}x + \frac{3}{2}$ $y = -2x + \frac{3}{2}$ $\therefore$ Slope $(m_A) = -2$	$-6x + 2y + 3 = 0$ $2y = 6x - 3$ $\frac{2y}{2} = \frac{6}{2}x - \frac{3}{2}$ $y = 3x - \frac{3}{2}$ $\therefore$ Slope $(m_B) = 3$

(b) **(i)** Statement: the point (−1, 4) is on the line $y = 3x + 7$.

Substitute the point (−1, 4) into the line $y = 3x + 7$.

$(4) = 3(-1) + 7$

$(4) = -3 + 7$

$4 = 4$

Therefore the statement is true.

(ii) Statement : the point (−3, 3) is not on the line $-3x - y - 6 = 0$.

Substitute the point (−3, 3) into the line $-3x - y - 6 = 0$.

$-3(-3) - (3) - 6 = 0$

$9 - 3 - 6 = 0$

$0 = 0$

Therefore the statement is false, as the point is on the line.

(iii) Statement: the point (0, −2) is on the line $x + 4y = 10$.

Substitute the point (0, −2) into the line $x + 4y = 10$.

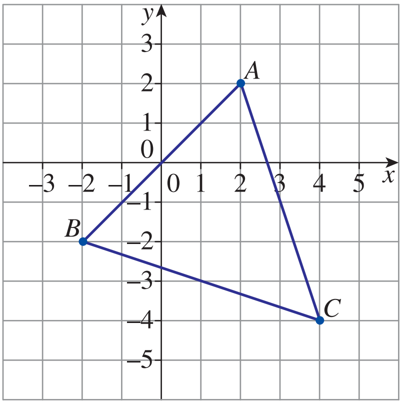
$(0) + 4(-2) = 10$

$0 - 8 = 10$

$-8 \neq 10$

Therefore the statement is false.

Q5 (a)



(b) To find the length of $|AB|$, label one of the points, say A, as (x_1, y_1) and the other as $B(x_2, y_2)$.

$$\begin{array}{cc} (x_1, y_1) & (x_2, y_2) \\ \Updownarrow\ \Updownarrow & \Updownarrow\ \Updownarrow \\ A(2,\ 2) & B(-2, -2) \end{array}$$

Substituting this information into the distance formula and simplifying:

$$\begin{aligned} |AB| &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-2 - 2)^2 + (-2 - 2)^2} \\ &= \sqrt{(-4)^2 + (-4)^2} \\ &= \sqrt{16 + 16} \\ &= \sqrt{32} = 4\sqrt{2} \end{aligned}$$

To find the length of $|AC|$, label one of the points, say A, as (x_1, y_1) and the other as $C(x_2, y_2)$.

$$\begin{array}{cc} (x_1,\ y_1) & (x_2,\ y_2) \\ \Updownarrow\ \Updownarrow & \Updownarrow\ \Updownarrow \\ A(2,\ 2) & C(4, -4) \end{array}$$

Substituting this information into the distance formula and simplifying:

$$\begin{aligned} |AC| &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(4 - 2)^2 + (-4 - 2)^2} \\ &= \sqrt{(2)^2 + (-6)^2} \\ &= \sqrt{4 + 36} \\ &= \sqrt{40} = 2\sqrt{10} \end{aligned}$$

To find the length of $|BC|$, label one of the points, say B, as (x_1, y_1) and the other as $C(x_2, y_2)$.

$$\begin{array}{ccc} (x_1, y_1) & & (x_2, y_2) \\ \Updownarrow\ \Updownarrow & & \Updownarrow\ \Updownarrow \\ B(-2, -2) & & C(4, -4) \end{array}$$

Substituting this information into the distance formula and simplifying:

$$\begin{aligned}|BC| &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(4 - (-2))^2 + (-4 - (-2))^2} \\ &= \sqrt{(6)^2 + (-2)^2} \\ &= \sqrt{36 + 4} \\ &= \sqrt{40} = 2\sqrt{10}\end{aligned}$$

(c) The triangle ΔABC is an isosceles triangle as two of the sides are the same length but the third is different.

Q6 (a) $a : -5x - 10y - 5 = 0$

$b : 4x - 2y - 8 = 0$

To find the slope of these lines, we can rearrange each equation $ax + by + c = 0$ to make y the subject of the equation in the form $y = mx + c$.

$a : -5x - 10y - 5 = 0$

$$-10y = 5x + 5$$
$$\frac{-10}{-10}y = \frac{5x}{-10} + \frac{5}{-10}$$
$$y = -\frac{1}{2}x - \frac{1}{2}$$
$$\therefore \text{ Slope } m_a = -\frac{1}{2}$$

$b : 4x - 2y - 8 = 0$

$$-2y = -4x + 8$$
$$\frac{-2}{-2}y = \frac{-4}{-2}x + \frac{8}{-2}$$
$$y = 2x - 4$$
$$\therefore \text{ Slope } m_b = 2$$

If the lines are perpendicular, then $m_a \times m_b = -1$

$$\left(-\frac{1}{2}\right) \times (2) = -1$$

Therefore the lines a and b are perpendicular to each other.

(b) $p : x - 3y + 12 = 0$

$q : x + 3y - 12 = 0$

To find the slope of these lines, we can rearrange each equation $ax + by + c = 0$ to make y the subject of the equation in the form $y = mx + c$.

$p : x - 3y + 12 = 0$

$$-3y = -x - 12$$
$$\frac{-3}{-3}y = \frac{-1}{-3}x - \frac{12}{-3}$$
$$y = \frac{1}{3}x + 4$$
$$\therefore \text{ Slope } m_p = \frac{1}{3}$$

$q : x + 3y - 12 = 0$

$$3y = -x + 12$$
$$\frac{3}{3}y = \frac{-1}{3}x + \frac{12}{3}$$
$$y = -\frac{1}{3}x + 4$$
$$\therefore \text{ Slope } m_q = -\frac{1}{3}$$

If the two lines are parallel, then $m_p = m_q$.

But $\frac{1}{3} \neq -\frac{1}{3}$.

Therefore the lines p and q are not parallel to each other.

(c) $s: 61 - 6y = 5x$

$t:\ 3y + 2x = 25$

Rearrange in the form $ax + by = c$

$s: 5x + 6y = 61$

$t: 2x + 3y = 25$

Multiply (s) by 2 and (t) by –5 and add the results, to eliminate x.

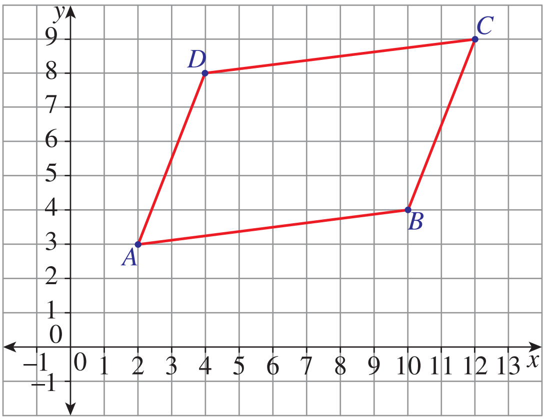
$$\begin{aligned} 2 \times (s) &\Rightarrow \quad 10x + 12y = 122 \\ -5 \times (t) &\Rightarrow \underline{-10x - 15y = -125} \\ &\qquad\qquad -3y = -3 \\ &\qquad\qquad \therefore\ y = 1 \end{aligned}$$

Substitute $y = 1$ in the equation for line t.

$$\begin{aligned} t: 2x + 3y &= 25 \\ 2x + 3(1) &= 25 \\ 2x + 3 &= 25 \\ 2x &= 22 \\ x = \frac{22}{2} &= 11 \end{aligned}$$

Therefore, the point of intersection is $(11, 1)$.

Q7 (a)



(b) Prove that $|AB| = |CD|$.

To calculate $|AB|$:

Label the points (x_1, y_1) and (x_2, y_2).

$$\begin{array}{cc} (x_1, y_1) & (x_2, y_2) \\ \Updownarrow\ \Updownarrow & \Updownarrow\ \Updownarrow \\ A(2, 3) & B(10, 4) \end{array}$$

Substitute this information into the distance formula and simplify.

$$\begin{aligned} |AB| &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(10-2)^2 + (4-3)^2} \\ &= \sqrt{(8)^2 + (1)^2} \\ &= \sqrt{64 + 1} \\ &= \sqrt{65} \end{aligned}$$

To calculate $|CD|$:

Label the points (x_1, y_1) and (x_2, y_2).

$$\begin{array}{cc} (x_1, y_1) & (x_2, y_2) \\ \Updownarrow\ \Updownarrow & \Updownarrow\ \Updownarrow \\ C(12, 9) & D(4, 8) \end{array}$$

Substitute this information into the distance formula and simplify.

$$\begin{aligned} |CD| &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(4-12)^2 + (8-9)^2} \\ &= \sqrt{(-8)^2 + (-1)^2} \\ &= \sqrt{64 + 1} \\ &= \sqrt{65} \end{aligned}$$

Therefore, $|AB| = |CD|$.

(Alternatively, work through the same process to prove that $|AD| = |BC|$.)

(c) To find the midpoint E of $[AC]$, label the points (x_1, y_1) and (x_2, y_2).

$$\begin{array}{cc} (x_1, y_1) & (x_2, y_2) \\ \Updownarrow\ \Updownarrow & \Updownarrow\ \Updownarrow \\ A(2, 3) & C(12, 9) \end{array}$$

Substitute this information into the midpoint formula:

$$\begin{aligned} E &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \\ &= \left(\frac{2 + 12}{2}, \frac{3 + 9}{2}\right) \\ &= \left(\frac{14}{2}, \frac{12}{2}\right) \\ &= (7, 6) \end{aligned}$$

To find the midpoint F of $[BD]$, label the points (x_1, y_1) and (x_2, y_2).

$$\begin{array}{cc} (x_1, y_1) & (x_2, y_2) \\ \Updownarrow\ \Updownarrow & \Updownarrow\ \Updownarrow \\ B(10, 4) & D(4, 8) \end{array}$$

Substitute this information into the midpoint formula:

$$\begin{aligned} F &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \\ &= \left(\frac{10 + 4}{2}, \frac{4 + 8}{2}\right) \end{aligned}$$

$= \left(\frac{14}{2}, \frac{12}{2}\right)$

$= (7, 6)$

As both the midpoints of $[AC]$ and $[BD]$ lie at (7, 6), the diagonals must bisect each other.

Q8 (a) Let $x = 0$ and find the corresponding y value.

Point is written as $(x, y) = (0, y \text{ value})$.

$5 + y - 2x = 0$

$5 + y - 2(0) = 0$

$5 + y = 0$

$\therefore y = -5$

Therefore, the line intersects the y-axis at $(0, -5)$.

Let $y = 0$ and find the corresponding x value.

Point is written as $(x, y) = (x \text{ value}, 0)$.

$5 + y - 2x = 0$

$5 + (0) - 2x = 0$

$5 = 2x$

$\therefore \frac{5}{2} = 2{\cdot}5 = x$

Therefore, the line intersects the x-axis at $(2{\cdot}5, 0)$.

(b) Rearrange the equation to be in the form $y = mx + c$:

$y = 2x - 5$

$\therefore$ Slope $= 2$

(c) (i) To calculate the slope of j, given that $m_l \perp m_j$.

$m_l \times m_j = -1$

$\therefore$ Slope of $j = -\frac{1}{2}$

(ii) The line j goes through the point (11, 6) and is perpendicular to the line l.

$\therefore$ Slope of $j = -\frac{1}{2}$

To work out the equation of j, use equation of a line in the form $y - y_1 = m(x - x_1)$.

$y - 6 = -\frac{1}{2}(x - 11)$

$2(y - 6) = -1(x - 11)$

$2y - 12 = -x + 11$

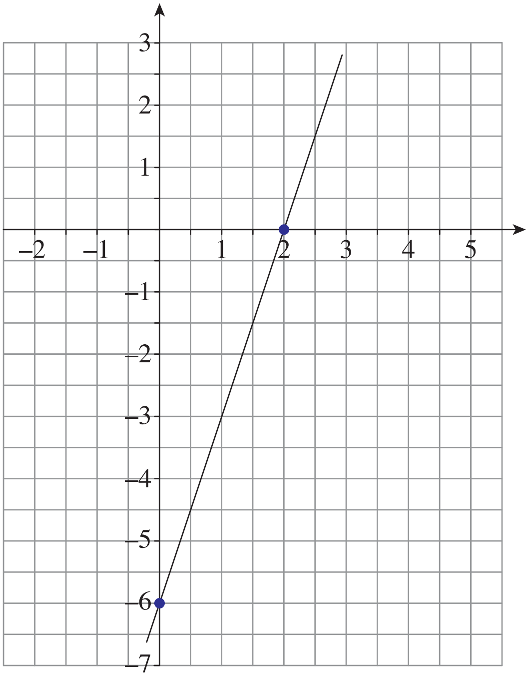
$x + 2y - 12 - 11 = 0$

$x + 2y - 23 = 0$

Q9 **(a)** Line 3, because 5 is the biggest number in front of x for any of the lines.

(b) Line 1 and Line 2, because they have the same slope (3).

(c) Work out the points of intersection of Line 1 with the x-axis and y-axis.



$y = 3x - 6$

Let $x = 0$ and solve for the y coordinate:

$y = 3(0) - 6$

$y = -6$

$\therefore$ Point of intersection on y-axis is (0, −6)

Now let $y = 0$ and solve for the x coordinate:

$0 = 3x - 6$

$6 = 3x$

$\frac{6}{3} = \frac{3x}{3}$

$2 = x$

$\therefore$ Point of intersection with the x-axis is (2, 0).

(d) The points of intersection on the axes are (0, 4), (2, 0).

Use the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ to work out the slope of the line.

$m = \frac{0 - 4}{2 - 0} = -\frac{4}{2} = -2$

Hence, given that the y-intercept is 4, the equation of the line is:

$y = -2x + 4$

Answer: Line 5

(e) Use two of the given points, say (7, 12) and (9, 20), to work out the slope of the line:

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{20 - 12}{9 - 7} = \frac{8}{2} = 4$

Now work out the equation of the line using the form $y - y_1 = m(x - x_1)$:

$y - 12 = 4(x - 7)$

$y - 12 = 4x - 28$

$y = 4x - 28 + 12$

$y = 4x - 16$

Answer: Line 6

To check this answer, substitute the given x values into the equation and check that the results are the given values of y.

$y = 4(7) - 16 = 12$

$y = 4(9) - 16 = 20$

$y = 4(10) - 16 = 24$

(f) Solve using simultaneous equations, by subtracting Line 4 from Line 6:

Line 6: $y = 4x - 16$

Line 4: $y = x - 7$

$0 = 3x - 9$

$9 = 3x$

$3 = x$

$\therefore\ y = 3 - 7 = -4$

Point of intersection = $(3, -4)$.

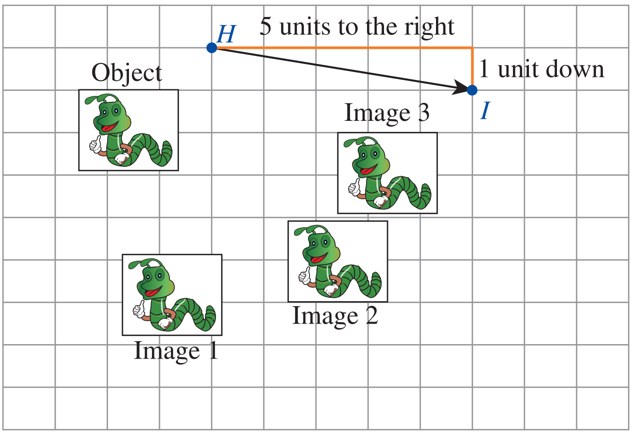
(g) Substitute the x value from part **(f)** into the equations for Line 4 and Line 6 and work out the corresponding y value.

Line 4	**Line 6**
$y = x - 7$	$y = 4x - 16$
$y = (3) - 7 = -4$	$y = 4(3) - 16 = 12 - 16 = -4$

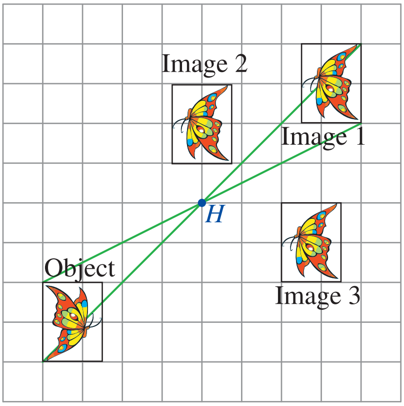
Both answers give the value of y as –4. This verifies the solution to part **(f)**.

Chapter 22 – Transformation Geometry

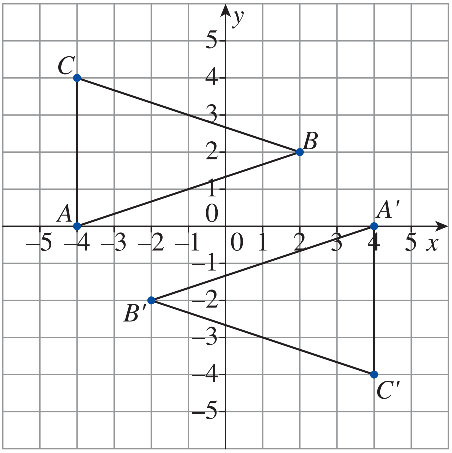
Q1 Under the given translation of 1 unit down and 5 units to the right, the correct image is Image 3.



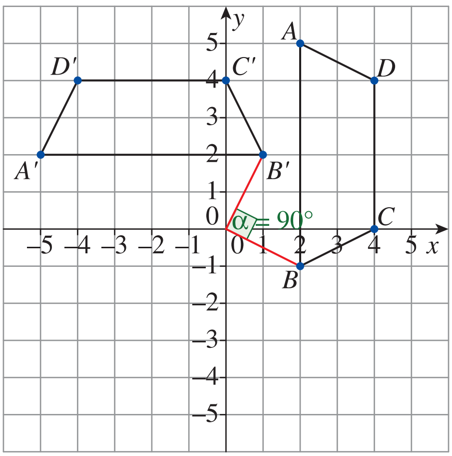
Q2 The Image of the object given under central symmetry in the point H is Image 1, as shown in the diagram.



Q3 **(a)**



(b)



Q4 **(a)** 3 axes of symmetry

(b) 6 axes of symmetry

(c) 1 axis of symmetry

Q5 (a) A translation can be used to map the line $\boxed{p \text{ or } r}$ onto the line $\boxed{r \text{ or } p.}$

(b) Axial symmetry in the y-axis can be used to map the line $\boxed{n \text{ or } s}$ onto the line $\boxed{s \text{ or } n.}$

(c) The line $\boxed{s \text{ or } n}$ is mapped onto itself under central symmetry in the point (0, 0).

Q6 A: Rotation of 90°[anti-clockwise] or Rotation of 270°[clockwise]

B: Axial symmetry

C: Translation or Rotation of 360°

Chapter 23 – Functions 1

Q1 A (a) The domain = {3, 5, 8, 15}

(b) The codomain = {7, 10, 13, 22, 43}

(c) The range = {7, 13, 22, 43}

(d) The set of couples for the function = {(3, 7), (5, 13), (8, 22), (15, 43)}

B (a) The domain = {1, 2, 3, 4}

(b) The codomain = {1, 4, 9, 16}

(c) The range = {1, 4, 9, 16}

(d) The set of couples for the function = {(1, 1), (2, 4), (3, 9), (4, 16)}

C (a) The domain = {−4, −1, 1, 4}

(b) The codomain = {1, 8, 16}

(c) The range = {1, 16}

(d) The set of couples for the function = {(−4, 16), (−1, 1), (1, 1), (4, 16)}

Q2 A does **not** represent a function as one input has no output.

B does **not** represent a function as one input has two different outputs.

C represents a function as each input has only one output.

D does **not** represent a function as two different inputs have more than one output.

Q3 To solve for a, given the input value of 11:

$h : x \to 10 - 2x$

$h(x) = 10 - 2x$

$h(11) = 10 - 2(11) = 10 - 22 = -12$

$\therefore a = -12$

To solve for b, given the output value of 24:

$h : x \rightarrow 10 - 2x$

$$h(b) = 10 - 2b = 24$$
$$-2b = 24 - 10$$
$$-2b = 14$$
$$\frac{-2b}{-2} = \frac{14}{-2}$$
$$b = -7$$

$\therefore b = -7$

Q4 (a) (i) $f(x) = 5x + 7$

$f(5) = 5(5) + 7 = 32$

(ii) $f(x) = 5x + 7$

$f(-5) = 5(-5) + 7 = -18$

(b) (i) $f(x) = 5x + 7$

$$f(q) = 5(q) + 7$$
$$= 5q + 7$$

(ii) $f(x) = 5x + 7$

$$f(q - 4) = 5(q - 4) + 7$$
$$= 5(q) - 5(4) + 7$$
$$= 5q - 20 + 7$$
$$= 5q - 13$$

Q5 (a) (i)

$h : x \rightarrow 2x - 4$

$h(x) = 2x - 4$

$h(-2) = 2(-2) - 4 = -8$

(ii)

$h : x \rightarrow 2x - 4$

$h(x) = 2x - 4$

$h(6) = 2(6) - 4 = 8$

(iii)

$h : x \rightarrow 2x - 4$

$h(x) = 2x - 4$

$h(12) = 2(12) - 4 = 20$

(b) (i)

$$h(x) = 2x - 4 = 2$$
$$2x = 2 + 4$$
$$2x = 6$$
$$\frac{2x}{2} = \frac{6}{2}$$
$$x = 3$$

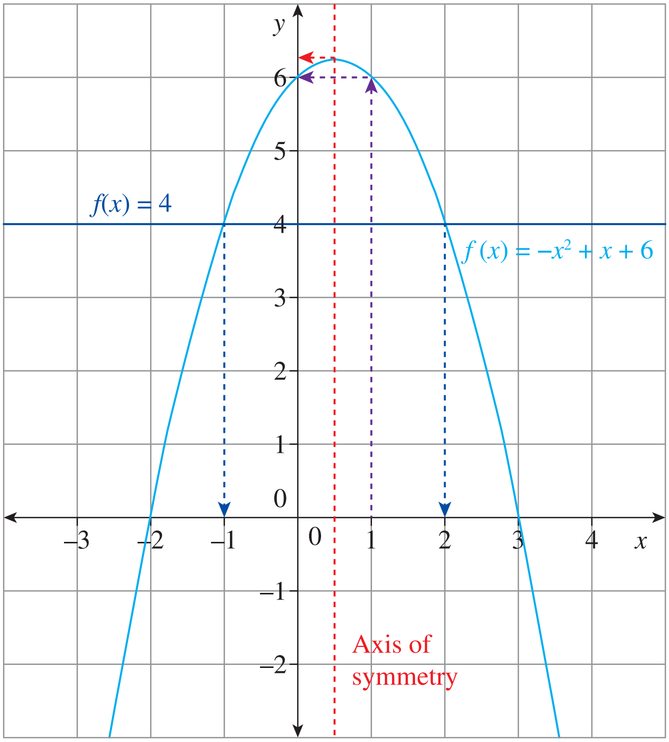
(ii)

$$h(x) = 2x - 4 = -14$$
$$2x = -14 + 4$$
$$2x = -10$$
$$\frac{2x}{2} = \frac{-10}{2}$$
$$x = -5$$

(iii)

$$h(x) = 2x - 4 = -3$$
$$2x = -3 + 4$$
$$2x = 1$$
$$\frac{2x}{2} = \frac{1}{2}$$
$$x = \frac{1}{2}$$

Q6



From the graph:

(a) Draw a horizontal line at the given value of $f(x) = 4$.

Where this line meets the curve, draw perpendicular lines to the x-axis to find the corresponding y values.

At $f(x) = 4$, the line intersects the curve at $x = -1$ and $x = 2$.

(b) Draw a vertical line from $x = 1$ until it meets the curve. Then, draw a horizontal line until it meets the y-axis to find the corresponding y value.

$f(1) = 6$

(c) The roots of the function are the solutions when $f(x) = x^2 + x + 6 = 0$.

The graph cuts the x-axis at $x = -2$ and $x = 3$.

(d) To estimate the maximum point of the function, draw a vertical line from the x-axis to the maximum point on the curve. Then draw a horizontal line to the y-axis to the corresponding y value.
Maximum point = (0·5, 6·2)

(e) To find the equation of the axis of symmetry of the function, draw a vertical line through the maximum or minimum point of the function.

The equation of the axis of symmetry of the graph is $x = 0{\cdot}5$.

Q7 At the point (1, 6), $f(1) = 6$.

$f(x) = -x^2 + ax + b$

$f(1) = -(1)^2 + a(1) + b = 6$

$-1 + a + b = 6$

$a + b = 7 \qquad (1)$

At the point (3, 4), $f(3) = 4$.

$f(x) = -x^2 + ax + b$

$f(3) = -(3)^2 + a(3) + b = 4$

$-9 + 3a + b = 4$

$3a + b = 13 \qquad (2)$

Subtract Equation (1) from Equation (2):

$$\begin{array}{rl} 3a + b = 13 & (2) \\ -\quad a + b = 7 & (1) \\ \hline 2a = 6 & \\ \therefore\ a = 3 & \end{array}$$

Substitute $a = 3$ into Equation (1) or (2) to solve for b.

$a + b = 7 \qquad (1)$

$3 + b = 7$

$b = 4$

Therefore the function is $f(x) = -x^2 + 3x + 4$.

Q8 C represents the y-intercept.

Let $x = 0$:

$f(0) = (0)^2 - 2(0) - 8$

$= -8$

Point $C = (0, -8)$

A and B represent the roots of the function.

Let $f(x) = 0$

$x^2 - 2x - 8 = 0$

$(x - 4)(x + 2) = 0$

$x - 4 = 0$ or $x + 2 = 0$

$x = 4$ or $x = -2$

The coordinates are $A(-2, 0)$ and $B(4, 0)$.

Q9 This graph does **not** represent a function, as we can see that some of the x-coordinates correspond to two y-coordinates.

Chapter 24 – Functions 2

Q1 **(a)** For each function, we need a minimum of two points that satisfy it.

For the function $f: x \rightarrow 4x + 8$:

- Solve for $(0, y)$ by setting $x = 0$, to find $y = f(0)$.

  $f(x) = 4x + 8$

  $f(0) = 4(0) + 8 = 8$

  The 1st point is (0, 8).

- Solve for $(x, 0)$ by setting $y = f(x) = 0$ and solve for x.

  $f(x) = 4x + 8 = 0$

  $4x + 8 = 0$

  $4x = -8$

  $x = -2$

  The 2nd point is (−2, 0).

For the function $g: x \rightarrow -x - 2$:

- Solve for $(0, y)$ by setting $x = 0$, to find $y = g(0)$

  $g(x) = -x - 2$

  $g(0) = -(0) - 2 = -2$

  The 1st point is (0, −2).

- Solve for $(x, 0)$ by setting $y = g(x) = 0$ and solve for x.

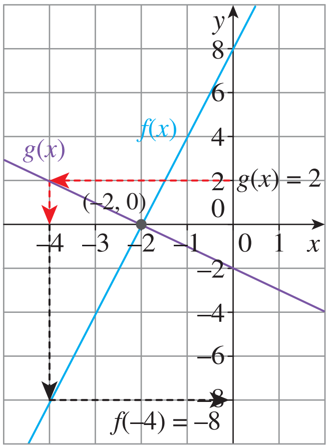
  $g(x) = -x - 2 = 0$

  $-x - 2 = 0$

  $x = -2$

  The 2nd point is (−2, 0).

- Plot these points on an x–y graph in the given domain.
- Draw in the line joining these points (extending the line in both directions).



(b) **(i)** $f(-4)$ means find the value of y when $x = -4$.

$f(-4) = -8$

(ii) $g(x) = 2$ means find where the function cuts the line $y = 2$.

$g(-4) = 2$

(iii) $f(x) = g(x)$ means find the point of intersection of the two functions.

The point of intersection is (−2, 0).

Q2 (a) For each function, we need a minimum of two points that satisfy it.

For the function $h : x \to -x + 3$:

- Solve for $(0, y)$ by setting $x = 0$, to find $y = h(0)$

  $h(x) = -x + 3$

  $h(0) = -(0) + 3 = 3$

  The 1st point is (0, 3).

- Solve for $(x, 0)$ by setting $y = h(x) = 0$ and solve for x.

  $h(x) = -x + 3 = 0$

  $-x + 3 = 0$

  $x = 3$

  The 2nd point is (3, 0).

For the function $g : x \to -x^2 + 4x + 3$:

Domain	$g : x \to -x^2 + 4x + 3$	**Range**	**Coordinates**
−2	$-(-2)^2 + 4(-2) + 3$	−9	(−2, −9)
−1	$-(-1)^2 + 4(-1) + 3$	−2	(−1, −2)
0	$-(0)^2 + 4(0) + 3$	3	(0, 3)
1	$-(1)^2 + 4(1) + 3$	6	(1, 6)
2	$-(2)^2 + 4(2) + 3$	7	(2, 7)
3	$-(3)^2 + 4(3) + 3$	6	(3, 6)
4	$-(4)^2 + 4(4) + 3$	3	(4, 3)
5	$-(5)^2 + 4(5) + 3$	−2	(5, −2)
6	$-(6)^2 + 4(6) + 3$	−9	(6, −9)

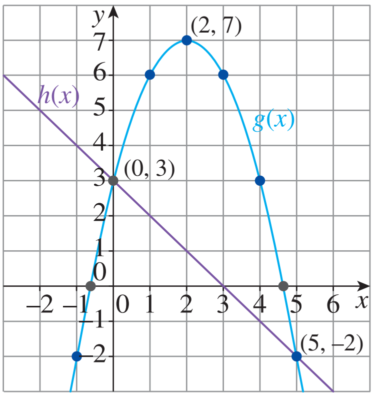
(b) **(i)** The maximum point of $g(x)$ is (2, 7).

(ii) $h(x) = g(x)$ means find the points of intersection of the two functions.

The points of intersection are (0, 3) and (5, −2).

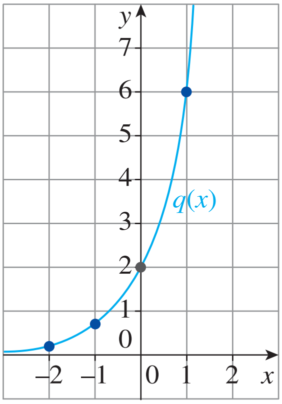
(iii) $h(x) \leq g(x)$ means find the values of x for which $h(x)$ is less than or equal to $g(x)$.

Answer: $0 \leq x \leq 5$



Q3 Set up and complete a table in the given domain.

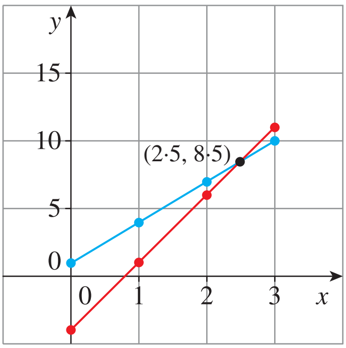
Domain	$q(x) = 2(3^x)$	Range	Coordinates
-2	$2(3^{-2})$	$\frac{2}{9}$	$\left(-2, \frac{2}{9}\right)$
-1	$2(3^{-1})$	$\frac{2}{3}$	$\left(-1, \frac{2}{3}\right)$
0	$2(3^0)$	2	(0, 2)
1	$2(3^1)$	6	(1, 6)



Q4 (a)

Domain	$f(x) = 5x - 4$	Range	Coordinates
0	$5(0) - 4$	-4	$(0, -4)$
1	$5(1) - 4$	1	(1, 1)
2	$5(2) - 4$	6	(2, 6)
3	$5(3) - 4$	11	(3, 11)

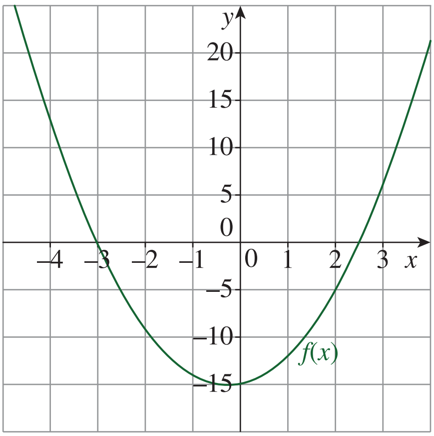
Domain	$g(x) = 3x + 1$	Range	Coordinates
0	$3(0) + 1$	1	(0, 1)
1	$3(1) + 1$	4	(1, 4)
2	$3(2) + 1$	7	(2, 7)
3	$3(3) + 1$	10	(3, 10)



(b) Point of intersection: (2·5, 8·5)

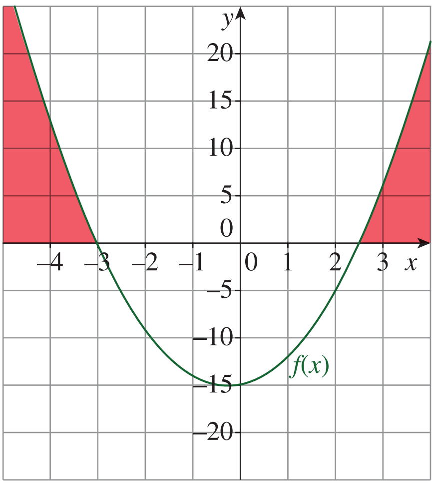
Q5 (a)

Domain	$f(x) = 2x^2 + x - 15$	Range	Coordinates
−4	$2(-4)^2 + (-4) - 15$	13	(−4, 13)
−3	$2(-3)^2 + (-3) - 15$	0	(−3, 0)
−2	$2(-2)^2 + (-2) - 15$	−9	(−2, −9)
−1	$2(-1)^2 + (-1) - 15$	−14	(−1, −14)
0	$2(0)^2 + (0) - 15$	−15	(0, −15)
1	$2(1)^2 + (1) - 15$	−12	(1, −12)
2	$2(2)^2 + (2) - 15$	−5	(2, −5)
3	$2(3)^2 + (3) - 15$	6	(3, 6)



(b) Minimum value of $f(x) \simeq -15{\cdot}1$

(c)



$f(x) \geq 0$ when $x \leq -3$ and $x \geq 2{\cdot}5$

Chapter 25 – Patterns

Q1 **(a)**    **(b)**    **(c)** 

Q2 **(i)** 11, 22, 33, 44, …

(a) $5(11) = 55$

(b) $11(10) = 110$

(c) $11n$

(d) Linear

(ii) 3, 7, 11, 15, …

(a) $3 + 4(5 - 1) = 3 + 4(4) = 3 + 16 = 19$

(b) $3 + 4(10 - 1) = 3 + 4(9) = 3 + 36 = 39$

(c) $3 + 4(n - 1)$

(d) Linear

(iii) 1, 4, 9, 16, …

(a) $(5)^2 = 25$

(b) $(10)^2 = 100$

(c) n^2

(d) Quadratic

(iv) 2, 4, 8, 16, …

(a) $2^5 = 32$

(b) $2^{10} = 1024$

(c) 2^n

(d) Exponential

Q3 **(a)** The pattern repeats in words as below:

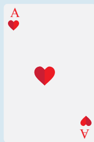
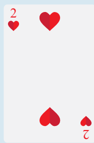
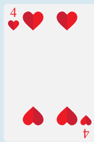
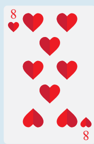
The first card in the pattern is the Ace of Hearts.

The second card in the pattern is the 2 of Hearts.

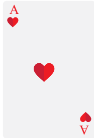
The third card in the pattern is the 4 of Hearts.

The fourth card in the pattern is the 8 of Hearts.

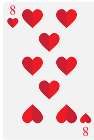
(b) The pattern repeats every 4th card, so the card pattern is found from the remainder:

Remainder	Card
$\frac{1}{4}$	
$\frac{2}{4}$	
$\frac{3}{4}$	
0	

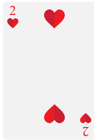
Hence, the 9th card in this pattern would be: $\frac{9}{4} = 2\frac{1}{4}$. The remainder is $\frac{1}{4}$, so the 9th card is the Ace of Hearts.



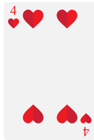
(c) Hence, the 16th card in this pattern would be: $\frac{16}{4} = 4$. There is no remainder, so the 16th card is the 8 of Hearts.



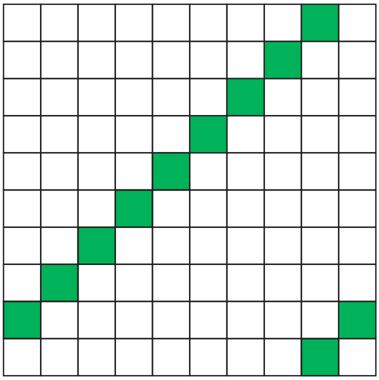
(d) Hence, the 202nd card in this pattern would be: $\frac{202}{4} = 50\frac{2}{4}$. The remainder is $\frac{2}{4}$, so the 202nd card is the 2 of Hearts.



(e) Hence, the 503rd card in this pattern would be: $\frac{503}{4} = 125\frac{3}{4}$. The remainder is $\frac{3}{4}$, so the 503rd card is the 4 of Hearts.



Q4 (a)



(b) 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99.

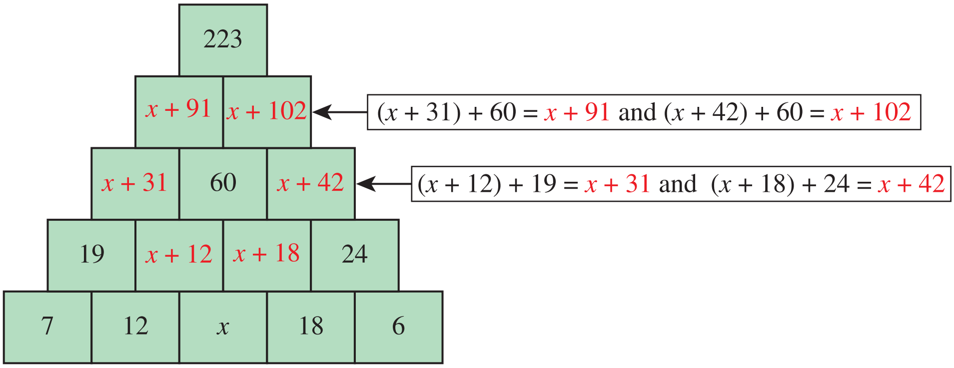
(c) The green squares are multiples of 9.

(d) $\frac{118}{9} = 13\frac{1}{9}$. Therefore, the square is white, as 118 is not a multiple of 9.

(e) $\frac{180}{9} = 20$. Therefore, the square is green, as 180 is a multiple of 9.

(f) $\frac{903}{9} = 100\frac{1}{3}$. Therefore, the square is white, as 903 is not a multiple of 9.

Q5 (a) Start from the bottom row of blocks and work up to the top of the pyramid.



(b) We can find the value of x, using either:

$(x + 91) + (x + 102) = 223$

$\Rightarrow\ 2x + 193 = 223$

$\Rightarrow\ 2x = 30$

$\Rightarrow\ x = 15$

or

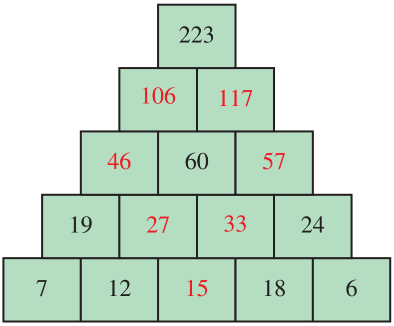
$(x + 12) + (x + 18) = 60$

$\Rightarrow\ 2x + 30 = 60$

$\Rightarrow\ 2x = 30$

$\Rightarrow\ x = 15$

(c)



Q6 (a) (i)

Pattern 1:	2		4		9		17		28
First difference: (Change)		+2		+5		+8		+11	
Second difference: (Change of change)			+3		+3		+3		

(ii) Pattern 2:

Pattern 2:	−1		2		5		8		11
First difference: (Change)		+3		+3		+3		+3	
Second difference: (Change of change)			0		0		0		

(iii) Pattern 3:

Pattern 3:	6		12		24		48		96
First difference: (Change)		+6		+12		+24		+48	
Second difference: (Change of change)			+6		+12		+24		

(b) **(i)** Pattern 1 is quadratic as the second difference is constant.

(ii) Pattern 2 is linear as the first difference is constant.

(iii) Pattern 3 is exponential as the general form is $3(2^n)$.
We know the pattern is exponential, as each consecutive term is found by multiplying the previous term by 2.

Q7 **(a)** $T_n = 2n^2 - 3$

$n = 1 \Rightarrow T_1 = 2(1)^2 - 3$
$\Rightarrow T_1 = 2 - 3$
$\Rightarrow T_1 = -1$
$n = 2 \Rightarrow T_2 = 2(2)^2 - 3$
$\Rightarrow T_2 = 8 - 3$
$\Rightarrow T_2 = 5$
$n = 3 \Rightarrow T_1 = 2(3)^2 - 3$
$\Rightarrow T_3 = 18 - 3$
$\Rightarrow T_3 = 15$

Answer: −1, 5, 15

(b) $T_n = 5n - 7$

$n = 1 \Rightarrow T_1 = 5(1) - 7$
$\Rightarrow T_1 = 5 - 7$
$\Rightarrow T_1 = -2$
$n = 2 \Rightarrow T_2 = 5(2) - 7$
$\Rightarrow T_2 = 10 - 7$
$\Rightarrow T_2 = 3$
$n = 3 \Rightarrow T_3 = 5(3) - 7$
$\Rightarrow T_3 = 15 - 7$
$\Rightarrow T_3 = 8$

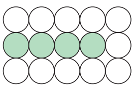
Answer: −2, 3, 8

(c) $T_n = 3(2^n)$

$n = 1 \Rightarrow T_1 = 3(2^1)$
$\Rightarrow T_1 = 3(2)$
$\Rightarrow T_1 = 6$
$n = 2 \Rightarrow T_2 = 3(2^2)$
$\Rightarrow T_2 = 3(4)$
$\Rightarrow T_2 = 12$
$n = 3 \Rightarrow T_2 = 3(2^3)$
$\Rightarrow T_3 = 3(8)$
$\Rightarrow T_3 = 24$

Answer: 6, 12 and 24.

Q8 **(a)**



(b)

Number of shaded discs	Number of white discs
1	5
2	**7**
3	**9**
4	**11**
5	**13**
6	**15**

(c) If we continue the pattern in the table:

Number of shaded discs	Number of white discs
1	5
2	**7**
3	**9**
4	**11**
5	**13**
6	**15**
7	**17**
8	**19**
9	**21**

We see that there are 9 shaded discs when there are 21 white discs.

(d) Let n = number of shaded discs.

Number of shaded discs	Number of white discs	First difference
1	5	
2	**7**	**+2**
3	**9**	**+2**
4	**11**	**+2**
5	**13**	**+2**
6	**15**	**+2**

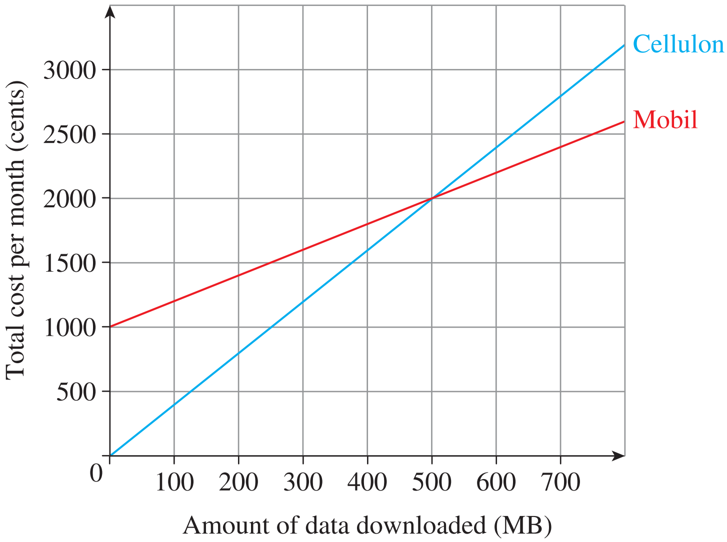
From the table we see that the common difference is +2. So we can state that there are 2 extra white discs for each extra shaded disc. When there are 0 shaded discs there would be 3 white discs (subtract 2 from 5); this is the starting value. Number of white discs = $3 + 2n$.

Chapter 26 – Graphs and Real-life Problems

Q1

Phone company	Total cost per month (cents)
Cellulon	$c(x) = 4x$ $c(0) = 4(0) = 0$ $c(700) = 4(700) = 2800$ Plot (0, 0) and (700, 2800).
Mobil	$m(x) = 1000 + 2x$ $m(0) = 1000 + 2(0) = 1000$ $m(700) = 1000 + 2(700) = 2400$ Plot (0, 1000) and (700, 2400).

(a)



(b) Answer: Cellulon

Using the graph: We can see that the line goes through (0, 0), so there is no cost if no data is used.

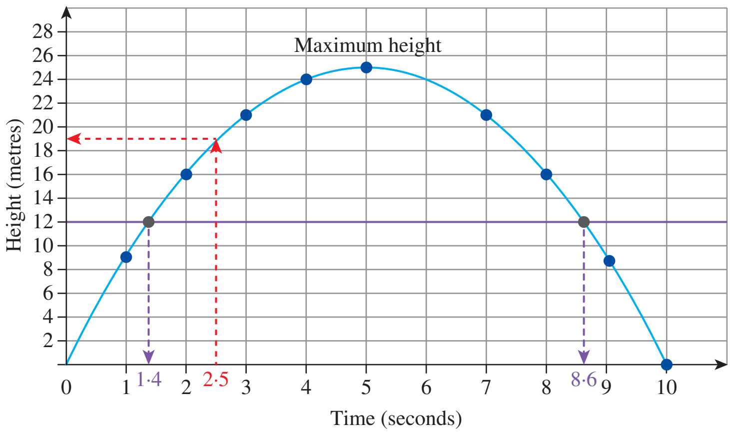
or

Using a formula: Given $c(0) = 0$; so there no cost if no data is used.

(c) The point of intersection of the graphs is (500, 2000).

(d) If the data used is less than 500 MB per month, Cellulon is cheaper. If the data used is more than 500 MB per month, Mobil is cheaper.

Q2



(a) From the graph, the height of the tennis ball after 2·5 seconds is 19 m.

(b) From the graph, the height of the tennis ball is 12 m above the ground at $t = 1{\cdot}4$ seconds and at $t = 8{\cdot}6$ seconds.

(c) From the graph, the maximum height of the tennis ball is 25 m.

Q3 (a) If the width of the flower bed is x m, an equation, in terms of x, for the length of the flower bed is derived as follows:

Total perimeter = 2(width) + 2(length) = 12 m

$\Rightarrow$ (width) + (length) = 6 m

$\Rightarrow x + l = 6$

Therefore $l = 6 - x$

(b) Proof that the area of the flower bed is given by $A(x) = 6x - x^2$:

Area = length × height

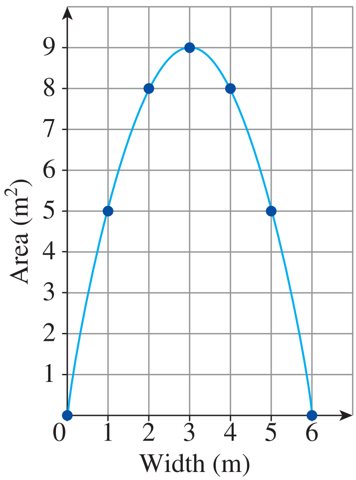
$= (6 - x) \times (x)$

$= 6x - x^2$

(c) To draw a graph to represent the area of the flower bed $A(x) = 6x - x^2$ as the width, x, changes from 0 m to 6 m, first construct a table for $A(x) = 6x - x^2$, then use the table to draw the graph.

Domain	$A(x) = 6x - x^2$	Range	Coordinates
0	$6(0) - (0)^2$	0	(0, 0)
1	$6(1) - (1)^2$	5	(1, 5)
2	$6(2) - (2)^2$	8	(2, 8)
3	$6(3) - (3)^2$	9	(3, 9)
4	$6(4) - (4)^2$	8	(4, 8)
5	$6(5) - (5)^2$	5	(5, 5)
6	$6(6) - (6)^2$	0	(6, 0)

(d) From the graph, a width of 3 m corresponds to the maximum area of 9 m^2.



Q4 **(a)** The car's highest speed is 60 km/h.

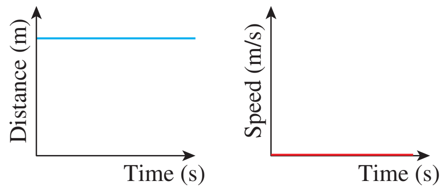
(b) To calculate how far the car travelled at its highest speed:

$$\begin{aligned}\text{Distance} &= \text{speed} \times \text{time}\\ &= 60 \times 10 \text{ min}\\ &= 60 \times \frac{1}{6} \text{ of an hour}\\ &= 10 \text{ km}\end{aligned}$$

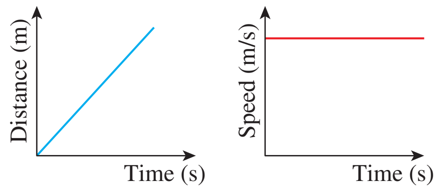
(c) From the graph, the car is travelling for 120 minutes (= 2 hours).

Q5

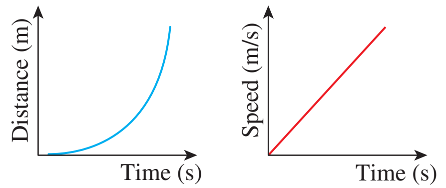
Distance–time graph **(a)** corresponds to speed–time graph **(iii)**.



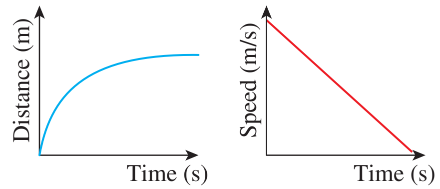
Distance–time graph **(b)** corresponds to speed–time graph **(i)**.



Distance–time graph **(c)** corresponds to speed–time graph **(iv)**.



Distance–time graph **(d)** corresponds to speed–time graph **(ii)**.



Q6 Container A corresponds to graph **(iii)**.

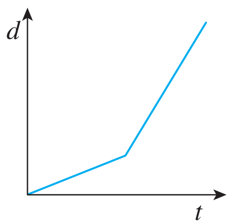
Container B corresponds to graph **(i)**.

Container C corresponds to graph **(ii)**.

Q7 Description **(a)** corresponds to graph **(ii)**.

(a) Object is moving at a slow pace, and then moves at a faster pace for a certain period of time.

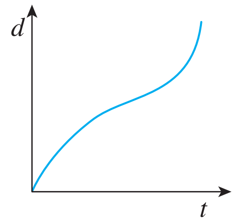
(ii)



Description **(b)** corresponds to graph **(iii)**.

(b) Object is moving at a fast pace, slows down and then speeds up again.

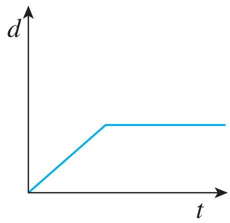
(iii)



Description **(c)** corresponds to graph **(i)**.

(c) Object is moving at a steady pace, then it comes to a stop and remains at rest for a certain period of time.

(i)



Chapter 27 – Trigonometry 1: Theory of Trigonometry

Q1 (a)

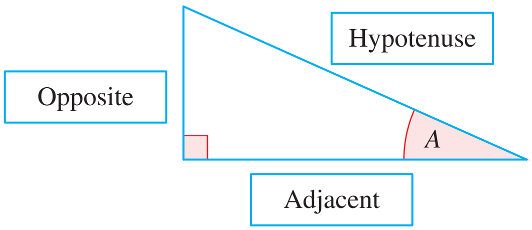
sin 25° = 0·423	cos 25° = 0·906	tan 25° = 0·466
sin 50° = 0·766	cos 50° = 0·643	tan 50° = 1·192

(b) **(i)** Maria was not correct. Values for the trigonometric ratios are not always less than 1. For example, tan 50° = 1·192.

(ii) Sharon was not correct, as doubling the angles did not result in the values for the trigonometric ratios doubling.

(iii) James was not correct, as the values for the trigonometric ratios did not always increase when the angle increased. For example, cos 25° = 0·906 and cos 50° = 0·643. In this case the angle increased and the value for the ratio decreased.

Q2 (a)



(b)

Trigonometric ratio	Ratio
$\sin A$	$\dfrac{\text{opposite}}{\text{hypotenuse}}$
$\cos A$	$\dfrac{\text{adjacent}}{\text{hypotenuse}}$
$\tan A$	$\dfrac{\text{opposite}}{\text{adjacent}}$

(c) $B = 35°$ and the opposite side is 12 cm. So to find the hypotenuse, use the sine ratio.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\Rightarrow \sin 35° = \frac{12}{\text{hypotenuse}}$$

$$\Rightarrow \text{hypotenuse} \times (\sin 35°) = 12$$

$$\Rightarrow \text{hypotenuse} = \frac{12}{\sin 35°}$$

$\Rightarrow$ hypotenuse = 20·92 cm = 21 cm to the nearest centimetre.

Q3 (a) Opposite = 12, adjacent = 9 and the hypotenuse = 15.

(i) $\sin A = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{12}{15} = \dfrac{4}{5}$

$\cos A = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{9}{15} = \dfrac{3}{5}$

$\tan A = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{12}{9} = \dfrac{4}{3}$

(ii) Any of the ratios found in part **(i)** can be used to find the measure (size) of the angle. They will all give the same value for A, as shown below.

$\sin A = \dfrac{4}{5} \Rightarrow A = \sin^{-1}\left(\dfrac{4}{5}\right) = 53{\cdot}13°$

$\cos A = \dfrac{3}{5} \Rightarrow A = \cos^{-1}\left(\dfrac{3}{5}\right) = 53{\cdot}13°$

$\tan A = \dfrac{4}{3} \Rightarrow A = \tan^{-1}\left(\dfrac{4}{3}\right) = 53{\cdot}13°$

(b) Opposite = $|YZ|$, adjacent = $|XY|$, hypotenuse = 13 and $|\angle YXZ| = 60°$.

(i) To find $|XY|$, use the cosine ratio.

$$\cos 60° = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{|XY|}{13}$$

$\Rightarrow 13\,(\cos 60°) = |XY|$

$\Rightarrow 6{\cdot}5 = |XY|$

$\Rightarrow |XY| = 6{\cdot}5$ units

(ii) To find $|YZ|$, use the sine ratio.

$$\sin 60° = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{|YZ|}{13}$$

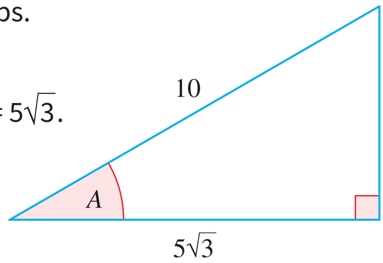
$\Rightarrow 13\,(\sin 60°) = |YZ|$

$\Rightarrow \dfrac{13\sqrt{3}}{2} = |YZ|$

$\Rightarrow |YZ| = 11{\cdot}26$ units, correct to 2 decimal places.

An alternative way of reaching the same solution is to use Pythagoras' Theorem and the result from part **(i)**.

Q4 To find the value of the angle A in the right-angled triangle shown (without using a calculator) follow these steps.



Write down all known information:

The hypotenuse = 10 and the adjacent side = $5\sqrt{3}$.

Therefore, we must use the trigonometric ratio: $\cos A = \dfrac{adj}{hyp}$

$\Rightarrow \cos A = \dfrac{5\sqrt{3}}{10}$

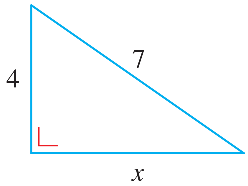
$\Rightarrow \cos A = \dfrac{\sqrt{3}}{2}$

$\Rightarrow A = \cos^{-1}\left(\dfrac{\sqrt{3}}{2}\right) = 30°$

Q5 (a) As the triangle PQR is an isosceles triangle then two of the sides are the same length.

Therefore the value of y can be either 8 or 11.

(b) Case 1: x lies on either the opposite or adjacent side.



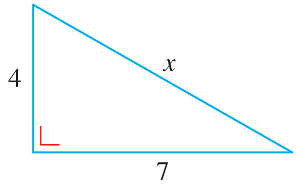
$4^2 + x^2 = 7^2$

$\Rightarrow x^2 = 49 - 16$

$\Rightarrow x^2 = 33$

$\Rightarrow x = \sqrt{33}$

Case 2: x lies on the hypotenuse side.



$4^2 + 7^2 = x^2$

$\Rightarrow x^2 = 49 + 16$

$\Rightarrow x^2 = 65$

$\Rightarrow x = \sqrt{65}$

Q6 (a) **Method 1:** Use Pythagoras' Theorem to solve for x.

$x^2 = 3^2 + 3^2 = 9 + 9$

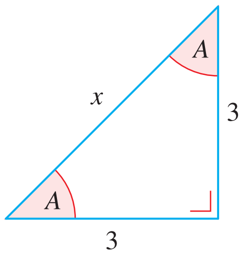
$\Rightarrow x = \sqrt{18}$

As lengths are positive, write down the positive value of $\sqrt{18}$ only.

$x = 3\sqrt{2}$

Method 2: Use trigonometric ratios to solve for x.

The triangle is isosceles, as the adjacent and opposite sides are the same length. Therefore, the angles opposite both sides are 45°.



We know: x = hypotenuse, opposite = 3, adjacent = 3 and $\angle A = 45°$.

Use the sine trigonometric ratio to find the value of x:

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\Rightarrow \sin 45° = \frac{3}{x}$$

$$\Rightarrow x = \frac{3}{\sin 45°}$$

$$x = \frac{3}{\left(\frac{1}{\sqrt{2}}\right)} = 3\sqrt{2}$$

(b) $y^2 + 1^2 = 3^2$

$\Rightarrow y^2 = 9 - 1$

$\Rightarrow y = \sqrt{8}$

As lengths are positive, write down the positive value of $\sqrt{8}$ only.

$y = 2\sqrt{2}$

(c) Let P = perimeter.

$P = 2x + 2y \Rightarrow 2(3\sqrt{2}) + 2(2\sqrt{2}) \Rightarrow 6\sqrt{2} + 4\sqrt{2} = 10\sqrt{2}$

Q7 **(i) (a)** is not true. For example, tan(250°) = 2·727. In this case the value of the trigonometric function is more than 1.

(ii) (b) is true. For example, sin(90°) = 0 but sin(180°) = −1. In this case the value of the trigonometric function did not double.

(iii) (c) is not true. For example, as cos(45°) = 0·7071, but cos(90°) = 0. In this case the value decreases as the angle increases.

(iv) Step 1: Construct an equilateral triangle with sides of length 2 cm, and then bisect (cut in two) the triangle to form two right-angled triangles.

Step 2: Find the length of the perpendicular height of the triangle.

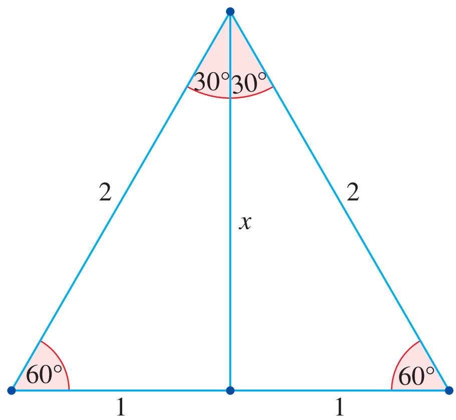
Let x denote the length of the perpendicular height.

Since both triangles are right-angled, they must satisfy Pythagoras' Theorem.

Thus, $1^2 + x^2 = 2^2$

$\Rightarrow x^2 = 4 - 1$

$\Rightarrow x = \sqrt{3}$



Step 3: In an equilateral triangle, the three angles are all 60°.
Using the result from Step 2:

In a right-angled triangle:

$$\sin 60° = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\Rightarrow \sin 60° = \frac{\sqrt{3}}{2}$$

Q8 **(a)** To find $|AB|$, in surd form, use Pythagoras' Theorem.

$|AC| = 2\sqrt{2}$ and $|BC| = 3\sqrt{3}$, and AB is the hypotenuse.

$|AB|^2 = |BC|^2 + |AC|^2$

$\Rightarrow |AB|^2 = (3\sqrt{3})^2 + (2\sqrt{2})^2 \Rightarrow 9(3) + 4(2) \Rightarrow 27 + 8 = 35$

The positive value is the only solution, as it is a length.

$|AB| = \sqrt{35}$ units

(b) Opposite $= 2\sqrt{2}$, adjacent $= 3\sqrt{3}$ and hypotenuse $= \sqrt{35}$.

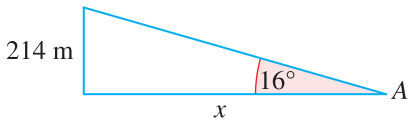
To find $|\angle ABC|$, correct to the nearest degree, any of the three trigonometric ratios can be used.

Option 1:	Option 2:	Option 3:
$\tan |\angle ABC| = \dfrac{\text{opposite}}{\text{adjacent}}$	$\sin |\angle ABC| = \dfrac{\text{opposite}}{\text{hypotenuse}}$	$\cos |\angle ABC| = \dfrac{\text{adjacent}}{\text{hypotenuse}}$
$\Rightarrow \tan |\angle ABC| = \dfrac{2\sqrt{2}}{3\sqrt{3}}$	$\sin |\angle ABC| = \dfrac{2\sqrt{2}}{\sqrt{35}}$	$\cos |\angle ABC| = \dfrac{3\sqrt{3}}{\sqrt{35}}$
$|\angle ABC| = \tan^{-1}\left(\dfrac{2\sqrt{2}}{3\sqrt{3}}\right)$	$|\angle ABC| = \sin^{-1}\left(\dfrac{2\sqrt{2}}{\sqrt{35}}\right)$	$|\angle ABC| = \cos^{-1}\left(\dfrac{3\sqrt{3}}{\sqrt{35}}\right)$
$|\angle ABC| = 28{\cdot}56°$	$|\angle ABC| = 28{\cdot}56°$	$|\angle ABC| = 28{\cdot}56°$

Hence $|\angle ABC| = 29°$ to the nearest degree.

Chapter 28 – Trigonometry 2: Applications of Trigonometry

Q1 First find the distance from the boat at point A to the cliff.



Label triangle A as shown.

Opposite side = 214 m and $\theta = 16°$.

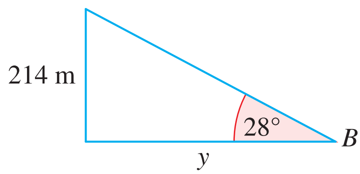
Required: the adjacent length x.

Use the trigonometric formula: $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$.

$\tan 16° = \dfrac{214}{x} \Rightarrow x = \dfrac{214}{\tan 16°} = 746{\cdot}306691$

$\Rightarrow x = 746{\cdot}31$ m

Then find how far it is from the boat at point B to the cliff.



Label triangle B as shown:

Opposite side = 214 m and $\theta = 28°$.

Required: the adjacent length y.

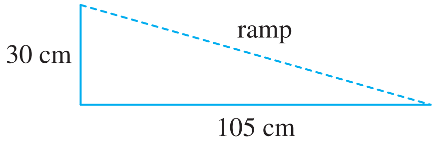
Use the trigonometric formula: $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$.

$\tan 28° = \dfrac{214}{y} \Rightarrow y = \dfrac{214}{\tan 28°} = 402{\cdot}475...$

$\Rightarrow y = 402{\cdot}48$ m

Hence, the distance the boat has sailed towards the cliff is:
$|AB| = x - y = 746{\cdot}31 - 402{\cdot}48 = 343{\cdot}83$ m $= 344$ m to the nearest metre.

Q2 The three steps are 10 cm high and 35 cm long. So the total height is 30 cm and the total length is 105 cm.



Use Pythagoras' Theorem to find the hypotenuse side.

$$\text{hypotenuse}^2 = 30^2 + 105^2$$
$$= \sqrt{900 + 11\,025}$$
$$= \sqrt{11\,925}$$
$= 109{\cdot}2$ cm, correct to one decimal place.

The ramp must have a length of $109{\cdot}2$ cm, correct to one decimal place.

Q3 (a) $\theta = 30°$, opposite $= 2$ m and hypotenuse $= x$ m.

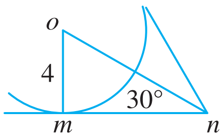
$$\sin 30° = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{2}{x} \Rightarrow \frac{1}{2} = \frac{2}{x} \Rightarrow x = 4 \text{ m}$$

(b) $\theta = 30°$, opposite $= 2$ m and adjacent $= \frac{y}{2}$ m.

$$\tan 30° = \frac{\text{opposite}}{\text{adjacent}} = \frac{2}{\left(\frac{y}{2}\right)} \Rightarrow y = 4\sqrt{3} = 6{\cdot}928... = 6{\cdot}93 \text{ m}$$

The length of the ceiling joist, correct to two decimal places is $6{\cdot}93$ m.

Q4 (a) As this is an equilateral triangle, each angle is 60°.



$\theta = 30°$and opposite $= |om| = 4$.

Required: the adjacent $= |mn|$

$$\tan 30° = \frac{\text{opposite}}{\text{adjacent}} = \frac{|om|}{|mn|} = \frac{4}{|mn|} \Rightarrow \frac{1}{\sqrt{3}} = \frac{4}{|mn|} \Rightarrow |mn| = 4\sqrt{3}$$

(b) Use Pythagoras' Theorem to find the hypotenuse side $|on|$:

$$\text{hypotenuse}^2 = 4^2 + \left(4\sqrt{3}\right)^2 \Rightarrow |on| = \sqrt{16 + 48} \Rightarrow |on| = \sqrt{64} \Rightarrow |on| = 8$$

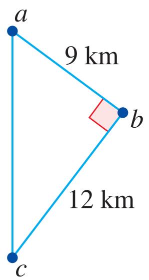
(c) The height of the equilateral triangle $= |on| + |om| = 8 + 4 = 12$.

(d) Total area = 6(area of the triangle Δmon)

$= 6 \times \frac{1}{2} \times$ base $\times$ perpendicular height

Area of the equilateral triangle $= 6 \times \frac{1}{2} \times 4\sqrt{3} \times 4 = 48\sqrt{3}$ units^2

Q5 **(a)** Use Pythagoras' Theorem to find the distance from a to c.



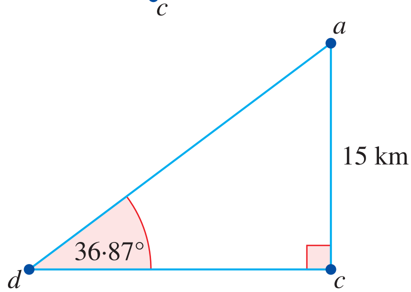
$$\text{hypotenuse}^2 = 12^2 + 9^2$$

$$\Rightarrow |ac| = \sqrt{144 + 81}$$

$$\Rightarrow |ac| = \sqrt{225}$$

$$\therefore |ac| = 15 \text{ km}$$

(b) $\theta = 36{\cdot}87°$ and opposite = 15 km.



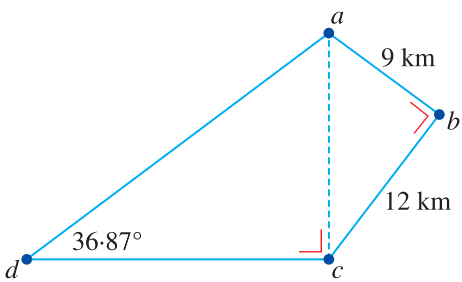
Required: the adjacent = $|cd|$

$$\tan 36{\cdot}87° = \frac{\text{opposite}}{\text{adjacent}} = \frac{15}{|cd|}$$

$$\Rightarrow |cd| = \frac{15}{\tan 36{\cdot}87°}$$

$$\Rightarrow |cd| = 19{\cdot}999... = 20 \text{ km}$$

(c) Use Pythagoras' Theorem to find the distance $|ad|$.



$$|ad|^2 = 20^2 + 15^2 = 400 + 225 = 625$$

$$\Rightarrow |ad| = \sqrt{625} = 25 \text{ km}$$

Therefore total length of the course = $|ab| + |bc| + |cd| + |da|$ = 9 + 12 + 20 + 25 = 66 km

Q6 This student is 1·6 m tall.

(a) Opposite = 1·6 m and adjacent = 3 m.

Required: angle A

$$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{1{\cdot}6}{3} = 0{\cdot}533... \Rightarrow A = \tan^{-1}(0{\cdot}5333) = 28{\cdot}07° = 28°$$

(b) $A = 28°$ and adjacent = 13 m.

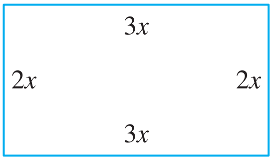
Required: opposite = h

$$\tan A = \frac{\text{opposite}}{\text{adjacent}} \Rightarrow \tan 28° = \frac{h}{13} \Rightarrow h = 13 \tan 28° = 6.9122 = 6{\cdot}9 \text{ m}$$

The height of the tree, correct to one decimal place is 6·9 m.

Chapter 29 – Applied Measure

Q1 First draw a sketch.



Given the perimeter of the rectangle is 200 cm:

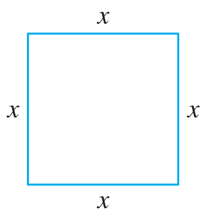
$3x + 2x + 3x + 2x = 200$ … state the relevant formula and let it equal the given value

$10x = 200$ … solve for the unknown value

$x = \frac{200}{10} = 20$

Area of the rectangle = length × breadth

$= 2x \times 3x = 2(20) \times 3(20) = 40 \times 60 = 2400$ cm^2



Q2 **(a)** Let the side of the square = x. Then

$4x = 96$

$\Rightarrow x = \frac{96}{4} = 24$ m

So area of the lawn = $x^2 = (24)^2 = 576$ m^2.

(b) As the answer is required in m^2, covert all measurements to metres.

Diameter = 75 cm

Therefore the cylinder has a radius of $= \frac{75}{2}$ cm = 37·5 cm

$= \frac{37{\cdot}5}{100}$ m = 0·375 m.

Curved surface area $= 2\pi rh$ … state the relevant formula

$= 2\pi(0{\cdot}375)(1)$ … substitute the known variables

$= 2(3{\cdot}14)(0{\cdot}375)(1) = 2{\cdot}355$ m^2

$= 2{\cdot}4$ m^2 to 1 d.p. … solve for the required value

(c) Area covered in nine revolutions $= 2{\cdot}4 \times 9 = 21{\cdot}6$ m^2.

Hence % completed by nine revolutions $= \frac{21{\cdot}6}{576} \times 100 = 3{\cdot}75\%$

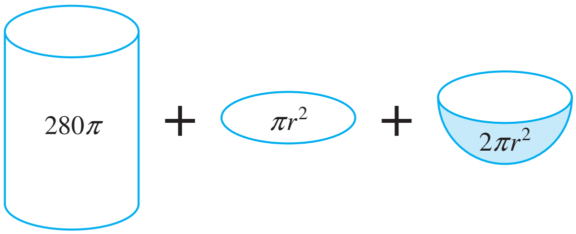
Q3 (a) Curved surface area = $2\pi rh$ ... state the relevant formula

$= 2\pi(7)(20)$ ... substitute the known variables

$= 280\pi$ cm^2 ... solve for the required value

(b) Let total surface area = T.

Then T = curved surface area of the cylinder + area of circle + curved surface area of hemisphere.

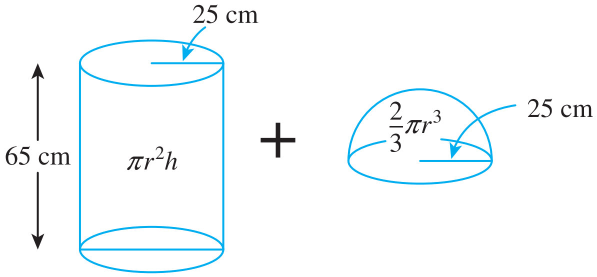


$T = 280\pi + \pi r^2 + 2\pi r^2$ ... state the relevant formulae

$= 280\pi + \pi(7)^2 + 2\pi(7)^2$ ... substitute the known variables

$= 280\pi + 49\pi + 98\pi = 427\pi$ cm^2 ... solve for the required value

Q4 Total volume of the shape = cylinder + hemisphere.



Total volume $= \pi r^2 h + \frac{2}{3}\pi r^3$ ... state the relevant formulae

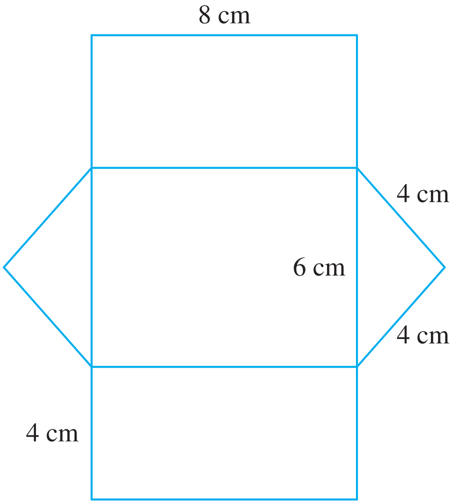
$= \pi(25)^2(65) + \frac{2}{3}\pi(25)^3$ ... substitute the known variables

$= 40\,625\pi + 10\,416{\cdot}66667\pi$ ... solve for the required value

$= 51041{\cdot}66667\pi = 160\,352{\cdot}125$ cm^3

Hence volume in litres $= \dfrac{160\,352{\cdot}125}{1000} = 160{\cdot}352125 = 160$ to the nearest litre.

Q5 Note: this diagram is not to scale.



Q6 (a) Volume = $l \times w \times h$ ... state the relevant formula

$(12)(5)h = 90$ ... substitute the known variables

$h = \frac{90}{60} = 1{\cdot}5$ cm ... solve for the required value

(b) Surface area $= 2(l \times w) + 2(w \times h) + 2(l \times h)$ ... state the relevant formula

$= 2(5)(12) + 2(12)(1{\cdot}5) + 2(5)(1{\cdot}5)$ ... substitute the known variables

$= 120 + 36 + 15 = 171$ cm^2 ... solve for the required value

(c) First convert the total mass from kg to g. **Note: 1 kg = 1000 g.**

$113{\cdot}4$ kg = 113 400 g

Mass of one block = $90 \times 8{\cdot}4$ g = 756 g

Hence number of blocks $= \frac{113\,400 \text{ g}}{756 \text{ g}} = 150.$

Q7 (a) Volume of the sphere $= \frac{4}{3}\pi r^3$ ... state the relevant formula

$\frac{4}{3}\pi r^3 = 36\pi$ ... substitute the known variables

$\frac{4}{3}r^3 = 36$ ... solve for the required value

$r^3 = \left(\frac{3}{4}\right)36 = \left(\frac{108}{4}\right) = 27 \Rightarrow r = \sqrt[3]{27} = 3$ cm.

(b) Let the rise in height $= h$. The volume of this part of the cylinder is therefore $\pi r^2 h$. So as the ball has a volume of 36π cm^3:

$\pi r^2 h = 36\pi$ ... state the relevant formula and let it equal the given value

$\pi r^2(2{\cdot}25) = 36\pi$ ... substitute the known variables

$r^2(2{\cdot}25) = 36$ ... solve for the required value

$$r^2 = \frac{36}{2{\cdot}25} = 16$$

Hence $r = \sqrt{16} = 4$ cm.

Q8 (a) From the diagram, the length of one side of the cube is $= \frac{24}{4} = 6$ cm.

(b) Total volume is made up of two cubes of side 6 cm and two spheres of diameter 6 cm.

$= 2(x)^3 + 2\left(\frac{4}{3}\pi r^3\right)$ ... state the relevant formula

$= 2(6)^3 + 2\left(\frac{4}{3}\pi (3)^3\right)$ ... substitute the known variables

$= 432 + 72\pi$ ... solve for the required value

$= 432 + 226{\cdot}1946711$

$= 658{\cdot}1945711$ cm^3 $= 658{\cdot}2$ cm^3, correct to 1 d.p.

(c) The total volume of the block before carving $= 24 \times 6 \times 6 = 864$ cm^3

The volume of wood carved away $= 864$ cm^3 $- 658{\cdot}2$ cm^3 $= 205{\cdot}8$ cm^3

Hence, the percentage of the original block of wood that was carved away

$$= \frac{205{\cdot}8}{864} \times \frac{100}{1} = 23{\cdot}82\%$$

Q9 (a) Area of the rectangle $= 966$ cm^2.

So $l \times h = 966$ cm^2

$42 \times h = 966$

$$\Rightarrow h = \frac{966}{42} = 23 \text{ cm.}$$

(b) Area of the triangle $= \frac{1}{2} \times l \times h = \frac{1}{2} \times 42 \times 23 = 483$ cm^2.