

LEAVING CERTIFICATE

MATHEMATICS

Includes a New and Enhanced Student Guide to Better Grades

2020





HIGHER LEVEL (WITH ANSWERS)

- Includes 2019 Exam Papers, Past Exam Papers and Edco Sample Paper
- O Answers to All Questions Included
- Free Online Worked Solutions visit www.e-xamit.ie
- O Exam Paper Analysis Chart Included
- Student Study Essentials
 - + Guide to better grades
 - + Online study hub
 - + Map your progress

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INCLUDES

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THE EDUCATIONAL COMPANY OF IRELAND

Leaving Certificate Mathematics Higher Level

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The phases refer to the gradual implementation of the new Project Maths syllabus up to 2014.

continued

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Completed (✓)

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ONLINE WORKED SOLUTIONS

Visit **www.e-xamit.ie** to access a bank of

Project Maths Tutorials, Sample Answers, Hints and Tips! (See codes inside)

^{*} These are sample questions from the SEC to support a change in the syllabus for Strand 1 for examination from 2015 onwards.

Student study essentials

Mathematics at Leaving Certificate

Higher Level

Exam Paper Analysis Chart

Paper 1	2019	2018 SEC	2017	2016	2015	2014 SEC	2014 SEC	2013 SEC	2012 SEC	2011 SEC	Edco
Topics	SEC	Paper	SEC	SEC	SEC	Paper	Sample	Paper	Paper	Paper	Sample A
	Paper		Paper	Paper	Paper	(Phase 3)	Paper	(Phase 3)	(Phase 3)	(Phase 2)	
							(Phase 3)				
Number	Q6b		Q8		Q6		Q8	Q3, Q4,	Q2	Q1a, b,	Q8
systems,								Q6d		Q5a, b, c	
Indices,										Q6	
Arithmetic,											
Financial											
Maths											
Algebra	Q1, Q2b,	Q1		Q2, Q4,	Q2, Q5a	Q3, Q7a	Q2, Q4	Q2	Q1, Q8a-e	Q1c, Q3,	Q2, Q3a,
	Q3a, b,			Q5a					Q8f = Trig.	Q4, Q5d	Q4b
	Q6a, Q9a										
Sequences,	Q7	Q2, Q5,	Q4	Q9	Q1	Q3b, c, Q6		Q9	Q4, Q9a		Q4a
Series		Q9									
Complex	Q5	Q4	Q2	Q1	Q4	Q2	Q1	Q1	Q3	Q2	Q1
Numbers											
Functions,	Q3c, Q4,	Q3, Q6b,	Q3, Q5,	Q5b, Q6,	Q3, Q5b,	Q4, Q5,	Q3, Q5, Q6,	Q5, Q6b, c,	Q5b, Q6,		Q5, Q6,
Calculus	Q8, Q9c	Q8	Q7	Q7	c, Q9	Q7b, Q8c,	Q9a, c	Q7, Q8	Q7, Q8c,		Q9
						Q9f			Q9b-f		
Functions,	Q2a, Q9b	Q6a, c, Q7	Q1, Q6,	Q3, Q8	Q7, Q8	Q1, Q8a, b,	Q7, Q9b	Q6a	Q5a		Q3b, Q7
Graphs			Q9			d, e, Q9a–e					

Paper 2	2019	2018	2017	2016	2015	2014 SEC	2014 SEC	2013 SEC	2012 SEC	2012 SEC	2011 SEC	2010 SEC
Topics	SEC	SEC	SEC	SEC	SEC	Paper	Sample	Paper	Paper	Sample	Paper	Paper
	Paper	Paper	Paper	Paper	Paper	(Phase 3)	Paper	(Phase 3)	(Phase 3)	Paper	(Phase 2)	(Phase 1)
							(Phase 3)			(Phase 1)		
Statistics	Q8	Q2, Q3,	Q2,	Q9	Q2	Q7, Q8b	Q2, Q7	Q2, Q7	Q5, Q7	Q2, Q7	Q2, Q7	Q2, Q7
		Q8	Q8a									
Probability	Q1, Q6	Q1	Q1,	Q5, Q6	Q1, Q8	Q3, Q8a	Q1	Q1	Q4	Q1	Q1	Q1, Q9A
			Q8b									
Geometry, Area,	Q5b	Q6, Q7	Q5, Q7	Q4	Q6	Q6	Q6, Q9*	Q6B, Q9	Q1, Q6	Q6	Q4, Q6	Q9B(a)
Volume												
Line	Q2	Q5a, b		Q1a	Q3	Q5	Q3	Q3	Q3**	Q3	Q3	Q6, Q8
Circle, Centroid,	Q3, Q5a	Q5c	Q3,	Q1b,	Q4	Q9	Q4	Q4, Q6A	Q2, Q3**	Q4	Q5	Q3, Q4
Circum-, In-,			Q4a	Q2								
Orthocentres												
Trigonometry,	Q4, Q7,	Q4, Q9	Q4b,	Q3,	Q5,	Q1, Q2,	Q5, Q8,	Q5, Q8	Q8	Q5, Q8	Q8	Q5,
Sectors, Period,	Q9		Q6,	Q7,	Q7,	Q4	Q9*					Q9B(b)
Range			Q9	Q8	Q9							

*Q9 overlaps two topics **Q3 overlaps two topics

 $\int (x-2)^2 dx =$

Some topics are interchangeable and can be examined on both paper 1 and paper 2.

Exam Hints and Tips

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Mathematics at Leaving Certificate

Guide to Better Grades

Higher Level

Structure of Papers

Paper 1

The topics generally examinable on this paper are:

- Strand 3 Number
- Strand 4 Algebra

• Strand 5	5 Functions/Calculus	
Section A	Concepts and skills	
	(6 questions)	150 marks
Section B	Contexts and applications	
	(3 questions)	150 marks
		1

All questions in sections A and B must be answered.

Paper 2

Two strands are generally examinable on this paper:

- Strand 1 Probability and statistics*
- Strand 2 Geometry and trigonometry
- Material from section 3.4, strand 3

Section A	Concepts and skills (6 questions)	150 marks
Section B	Contexts and applications (3 questions)	150 marks

All questions in sections A and B must be answered.

Some topics are interchangeable and can be examined on both Paper 1 and Paper 2.

*In Strand 1, material dealing with inferential statistics was previously deferred but has been examinable since 2015.

Topics will not appear in a set question each year and students should expect that questions may sometimes involve skills and knowledge from more than one strand.

Project Maths: General Assessment criteria (quote from from the syllabus):

"A high level of achievement in Mathematics is characterised by a demonstration of a thorough knowledge and comprehensive understanding of mathematics as described by the learning outcomes associated with each strand. The student is able to make deductions with insight even in unfamiliar contexts and can move confidently between different forms of representation. When investigating challenging problems, the student recognises pattern structures, describes them as relationships or general rules, draws conclusions and provides justification or proof. The student presents a concise, reasoned justification for the method and process and, where appropriate, considers the range of approaches which could have been used, including the use of technology".

Working Procedures

Students are advised to show all their work clearly at all times. It should be noted that, normally, wrong answers without work are worthless. Thus, work leading to a solution should be clearly presented wherever possible, even if the solution appears obvious to you!

Give the answer in the required form, e.g. to a particular number of decimal places. Always include the appropriate unit.

Common Examination Question Instructions

DIFFERENTIATE FROM FIRST PRINCIPLES: Must use the $y+\Delta y$ or f(x+h) methods. Use of formulae or rules to differentiate when 'first principles' is in the question would yield at most the attempt mark; more usually zero marks.

DIFFERENTIATE: (No mention of 'first principles') usually implies the use of at least one of the rules – product, quotient or chain rules.

DRAW: The graph of a curve or a function, using graph paper with properly identified axes and scales.

EVALUATE: Usually used in integration questions where limits are given. Failure to insert limits or incorrectly substituting limits would be penalised.

EXPRESS IN THE FORM...: This implies some simplification with the answer to be in a required format. When answers are required in a particular form, full marks will not be allocated if the answer is not in the desired form. For example, an answer required in the form a^x must be presented as 2⁴ and not as 16.

For example: Differentiate $5sin^23x$ and express your answer in the form a sin bx.

Differentiating $5sin^23x$ gives 10sin3x cos 3x (not in the required form) = 5sin 6x.

FIND THE ROOTS OF: = Solve, usually for x.

FIND THE SOLUTION SET OF: = Solve.

HENCE ...: This indicates a link between what has just been completed and the next part of the question. When "Hence ..." is used, candidates must use the first result to work in the next part; otherwise a penalty may apply. Often, however, the question is worded as "Hence, or otherwise, ..." This means there is a link between the part just completed and the next part, but candidates may attempt this next part without using the result of the first part. It is usually easier to follow the lead given by the first part.

INVESTIGATE: Used to check if a certain statement is true or false.

For example: Investigate whether or not 3 is a root of $x^3 - x^2 - 10x + 12 = 0$.

PROVE: As applied to results, the proofs of which can be asked, as in trigonometrical formulae, De Moivre, coordinate geometry formulae, differentiation formulae. Can also be applied to inequalities, such as:

prove $a^2 + b^2 \ge 2ab$ for all $a, b \in \mathbb{R}$

PROVE BY INDUCTION: Indicates that the method of induction must be used in order to be considered for full marks.

SHOW: Any correct method of showing a statement is true is acceptable except using particular values unless their use is specifically mentioned in the question.

For example: Show that the equation $x^3 - x^2 - 1 = 0$ has a root between 1 and 2 requires these values to be substituted.

Show that $\frac{1}{n} - \frac{1}{n+1} = \frac{1}{n(n+1)}$

for all $n \in N$ – specific values of n not acceptable as a proof.

SIMPLIFY: Means to carry out a task such as removing brackets, multiplying, dividing and to make the result as simple as possible.

For example: Simplify (3 + 2i)(4 - i) = 12 - 3i + 8i - 2i2 is not finished.

SKETCH: A sketch is a non-accurate drawing of a situation. For example, when asked to sketch the graph of a function, what is required is a 'rough' indication of the orientation and location of the graph, often using information found in earlier parts of the question.

VERIFY: Involves substituting value(s) or expressions into an equation or inequality to show the statement is true.

Exam Procedure: Commonly Asked Questions

- Q. What can and should I bring into the exam centre?
- A. (i) A calculator that you know how to use.(ii) Pens and pencils plus an eraser (to be used sparingly).
- **Q.** What should I get from the superintendent in the exam centre?
- **A.** The Maths *formulae and tables* booklet which contains formulae and tables for all topics, graph or squared paper, and extra paper to write on should you need it.
- **Q.** What is the first thing I should do when I get the exam paper?
- **A.** Ask for a copy of the Maths *formulae and tables* booklet and make sure you know where the formulae and rules are for the various topics. Then read through the paper, looking carefully at each question, and marking those you think are easy.

- Q. Do I write in pen or pencil?
- **A.** Write in blue/black pen and only use pencil for graphs or constructions.
- **Q.** Do I have to start at Question 1?
- A. No. We recommend that you start with your best question and follow this with your next best and so on. That way, when you come to harder parts, you will be more confident in tackling them. Remember, it is very important that you try not to leave any parts unattempted, no matter how difficult you think it is.
- **Q.** If I decide that the solution I have written down is wrong, what should I do?
- A. Do NOT use Tippex or cross it out. Simply draw a line and start again. If you attempt the same part of a question more than once, each attempt is marked and the highest of the marks gained in the various attempts is the final mark awarded.
- Q. Is 'rough work' of any value?
- A. All work handed in will be marked. Candidates often gain marks from work done in rough work so do not erase it. It sometimes also helps the examiner decide how you arrived at a solution.
- Q. Do I have to do all parts of a question one after the other?
- **A.** No. As long as you clearly identify which part of which question you are answering, the order in which you answer questions or even parts of questions is up to you.
- **Q.** How are the papers marked?
- A. Candidates will be marked according to a number of marking scales, depending on the type of response that is anticipated. In the case of scales labelled "A", candidate responses are divided into two categories: correct and incorrect. In the case of scales labelled "B", candidate responses are divided into three categories: correct, partially correct, and incorrect. Scales "C", "D", and "E" have four, five and six categories respectively. The scales and the marks that they generate are summarised in the table on the next page.



How to achieve to the best of your ability:

- **Read guestions carefully**. The wording on examination papers is carefully thought out and is concise and deliberate, so it is easy to miss or misread a critical piece of information. Read the question carefully before you begin answering it.
- Show all your work. Partial credit will be awarded for any work of merit.
- Attempt all parts of the questions you are doing. . The examiner will always search for merit in what you write. But if you write nothing, you cannot get any marks.
- If you make more than one attempt at a guestion, make • it clear which attempt is your final version. However, you should also ensure that your other attempts remain legible. In most circumstances, you will get credit for your best attempt, even if it has been cancelled in favour of another.
- Even if you are not asked to **draw a diagram**, it can often be a very helpful first step. You may gain some credit for the diagram. More importantly, the way forward with the problem very often becomes much clearer when the given information is presented on a diagram.
- Do not be put off or upset if a problem is not working out. Some problems are intended to be challenging. When a question is not what you expected, then you will be well rewarded for exploring the problem in a reasoned way, even if you do not fully solve the problem.
- Communicate your thinking as clearly as possible, whether you are solving a mathematical problem or offering a textbased answer.
- Ensure that you are thoroughly familiar with your own calculator and capable of using it efficiently and intelligently. Make sure that it has a sufficient range of features to meet your needs during the examination.

Scale label	A	В	С	D	E
No of categories	2	3	4	5	6
5 mark scale	0, 5	0, 2, 5	0, 2, 4, 5		
10 mark scale	0, 10	0, 5, 10	0, 3, 7, 10	0, 2, 5, 8, 10	
15 mark scale	0, 15	0, 7, 15	0, 5, 10, 15	0, 4, 7, 11, 15	
20 mark scale	0, 20	0, 10, 20	0, 7, 13, 20	0, 5, 10, 15, 20	
25 mark scale		0, 12, 25	0, 8, 17, 25	0, 6, 12, 19, 25	0, 5, 10, 15, 20, 25

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the body of the scheme, where necessary.

Marking Scales - Level Descriptors	C-scales (four categories)
A-scales (two categories)	 response of no substantial merit (no credit)
incorrect response (no credit)	 response with some ment (low partial credit) almost correct
 correct response (full credit) 	response (high partial credit)
B-scales (three categories)	correct response (full credit)
 response of no substantial merit (no credit) 	D-scales (five categories)
 partially correct response (partial credit) 	 response of no substantial merit (no credit)
 correct response (full credit) 	• response with some merit (low partial credit)
 response about half-right (middle paitial credit) 	• response almost half-right (lower middle partial credit)
 almost correct response (high partial credit) 	• response more than half-right (upper middle partial credit)
correct response (full credit)	• almost correct response (high partial credit)
E-scales (six categories)	correct response (full credit)
 response of no substantial merit (no credit) 	
• response with some merit (low partial credit)	

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Sample A Paper 1 Paper 1 Sample Paper 1 2014 SEC Phase 3) Phase 3) Paper 1 Paper 1 Paper 1 Paper 1 2 hrs 30 mins 5-10 mins LEAVING CERTIFICATE MATHEMATICS (HL) Section B (Contexts and Applications) 150 marks (answer all questions) Section A (Concepts and Skills) 150 marks (3 questions) Paper 1 (300 marks) Read over paper. Q.1 (25 marks) Q.2 (25 marks) Q.3 (25 marks) Q.5 (25 marks) Q.6 (25 marks) Q.4 (25 marks) Q.7 0. 8

Tick each question as you complete it and again once you have finished an entire exam paper.

(Continued)

Remember

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Exam Complete

Q.9

- Marks will be lost if all necessary work is not clearly shown.
- Answers should include the appropriate units of measurement, where relevant. •

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